Panel 1

Last Time

Dot Product: $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

Geometrically: $\dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \cos(\theta)$

Projection of $\vec{b}$ onto $\vec{a}$: $proj_{\vec{a}}(\vec{b}) = \dfrac{\vec{a} \cdot \vec{b}}{\|a\|^2} \vec{a}$

Length of $proj_{\vec{a}}(\vec{b})$: $comp_{\vec{a}}(\vec{b}) = \dfrac{|\vec{a} \cdot \vec{b}|}{\|a\|}$

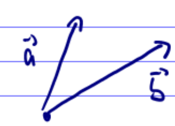
Properties of dot product: $\vec{a} \cdot \vec{a} = \|a\|^2$

1

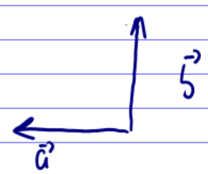
Panel 2

Picture Problems

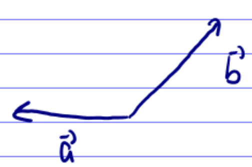
$$\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \cos\theta \iff \vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cos(\theta)$$



$\vec{a} \cdot \vec{b}$ — positive / zero / negative



$\vec{a} \cdot \vec{b}$ — positive / zero / negative

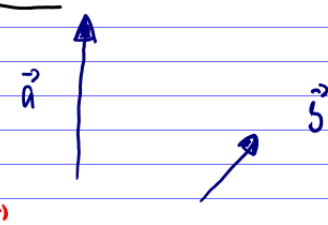


$\vec{a} \cdot \vec{b}$ — positive / zero / negative

2

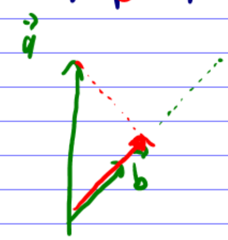
Panel 3

More picture Problems



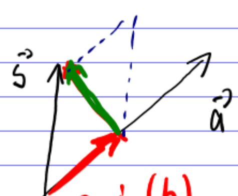
find $proj_{\vec a}(\vec b) = \frac{\vec a \cdot \vec b}{\|a\|^2}\vec a$

$proj_{\vec b}(\vec a) = \frac{\vec a \cdot \vec b}{\|b\|^2}\vec b$



3

Panel 4



$proj_a(b)$

It looks like

$proj_{\vec a}(\vec b) - \vec b \perp \vec a$

Prove it:

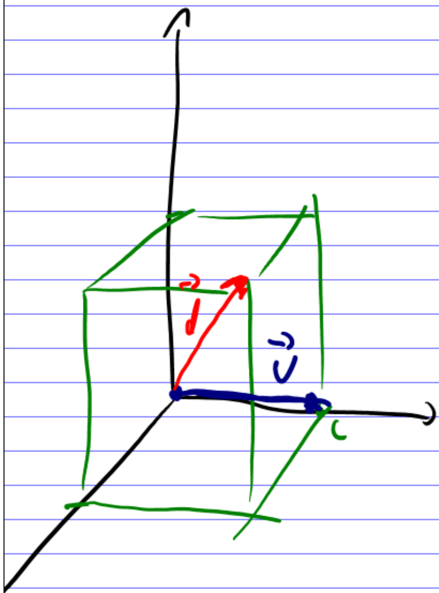
$$\left(\frac{\vec a \cdot \vec b}{\|a\|^2}\vec a - \vec b\right) \cdot \vec a =$$

$$\frac{\vec a \cdot \vec b}{\cancel{\|a\|^2}}\cancel{\vec a \cdot \vec a} - \vec b \cdot \vec a$$

$$\vec a \cdot \vec b - \vec b \cdot \vec a = 0$$

4

Panel 5



$\vec{v} = \langle 0, c, 0 \rangle$

$\vec{d} = \langle c, c, c \rangle$

$$\frac{\vec{v} \cdot \vec{d}}{\|v\| \|d\|} = \frac{c^2}{c\sqrt{3}c} = \frac{1}{\sqrt{3}} = \cos(\theta)$$

$\theta = \arccos\left(\frac{1}{\sqrt{3}}\right) = 57.^\circ$

or $54.^\circ$

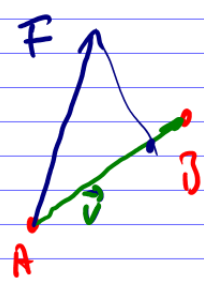
5

Panel 6

$F = 8i - 6j + 9k$

$(0, 10, 8)$ to $(6, 12, 20)$

A B



$\| (\mathrm{proj}_{\vec{v}} \vec{F}) \| \cdot \| \vec{v} \|$

$$\left\| \frac{\vec{F} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} \right\| \cdot \| \vec{v} \| = | \vec{F} \cdot \vec{v} | =$$

$\vec{v} = \langle 6, 2, 12 \rangle$

$= \langle 8, -6, 9 \rangle \cdot \langle 6, 2, 12 \rangle = \underline{\underline{144}}$

6

Panel 7

#13) center $\left(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}, \frac{a_3+b_3}{2}\right)$

radius $\frac{1}{2}\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$

7

Panel 8

Quiz #2

Name: ____________

① Find the dot product $\langle 3,-2,1\rangle \cdot \langle 1,2,2\rangle$

② Which vector is perpendicular to $\langle 3,-2,1\rangle$:

a) $\langle 1,1,1\rangle$

b) $\langle 2,4,2\rangle$

8

Panel 9

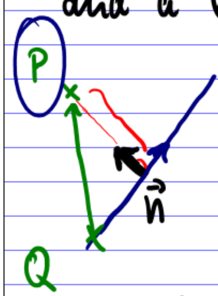
(3) Find the projection of $\langle 3,-1,-2\rangle$ onto $\langle 3,3,1\rangle$

(4) Find the work done by the force $\vec{F} = \langle 2,1,1\rangle$ that moves an object in the direction $\langle 1,1,1\rangle$ for 10 meters.

9

Panel 10

Find a formula for the distance between $P(x_0, y_0)$ and a line $ax + by + c = 0$.



1. Take Q any point on the line
2. Find $\vec{n}$ perp. to line
3. Project PQ onto $\vec{n}$ and find length

$Q(0, -\frac{c}{b})$ Direction of line $(0, -\frac{c}{b})$ to $(-\frac{c}{a}, 0)$ is $(\frac{c}{a}, -\frac{c}{b}) \cdot \frac{ab}{c} = (b, -a)$

$\vec{n} = (a, b)$

3. $$\frac{|\vec{PQ} \cdot \vec{n}|}{\|\vec{n}\|} = \frac{\langle x_0, y_0 + \frac{c}{b}\rangle \cdot (a, b)}{\sqrt{a^2 + b^2}} = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

10

Panel 11

So: Add / Subtract vector → vector
Dot product of vectors → scalar

Def (Cross Product) $\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$

$$\vec{a} \times \vec{b} = \langle a_2 b_3 - a_3 b_2, -(a_1 b_3 - a_3 b_1), a_1 b_2 - a_2 b_1 \rangle$$

Things to note

$a \cdot b$ easy but "only" scalar

$a \times b$ hard but a vector

11

Panel 12

How to memorize the cross product:

$$\vec{a} \times \vec{b} = \langle \underline{a_2 b_3 - a_3 b_2}, \underline{a_3 b_1 - a_1 b_3}, \underline{a_1 b_2 - a_2 b_1} \rangle$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i(a_2 b_3 - a_3 b_2) + j(-1)(a_1 b_3 - a_3 b_1)$$
$$k(a_1 b_2 - a_2 b_1)$$

Ex: $\langle 1, 3, 4 \rangle \times \langle 2, 7, -5 \rangle =$

$$\begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{vmatrix} = \langle \underset{i}{-15 + 28}, \underset{j}{-(-5 - 8)}, \underset{k}{+ -6} \rangle$$

12

Panel 13

$\langle 2,-1,2\rangle \times \langle 0,3,-1\rangle$

$$\begin{vmatrix} i & j & k \\ 2 & -1 & 2 \\ 0 & 3 & -1 \end{vmatrix} \quad \langle (-1)(-1)+(3)(2), -[(2)(-1)-(0)(2)], (2)(3)-(0)(-1)\rangle$$

$= \langle -5, +2, 6\rangle$

$\langle 1,0,0\rangle \times \langle 0,1,0\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \quad \langle (0)(0)-(0)(1), -((1)(0)-(0)(0)), (1)(1)-(0)(0)\rangle$$

$\langle 0, 0, 1\rangle$

13

Panel 14

Properties: (1) $\vec{a} \times \vec{a} =$

(2) $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

Prove: (1)

$$\begin{array}{ccc} i & j & k \\ a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \end{array}$$

(2) $(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$

HW

14

Panel 15

Cross out the expressions that do not make sense. For the rest, is the answer a vector or a scalar?

$a \cdot (b \times c)$	$(a \cdot b) \times c$
$\cancel{(a \cdot b) \cdot c}$	$(a \times b) + c$
$\cancel{a \times (b \cdot c)}$	$(a \cdot b) \times (c \cdot d)$
$\lVert a \rVert (b \cdot c)$	$(a \cdot b) + c$
$a \times (b \times c)$	$(a \times b) \cdot (c \times d)$
$a \cdot (b + c)$	$\lVert a \rVert (b \times c)$
$(a \times b) + c$	$\lVert a \times b \rVert$

15