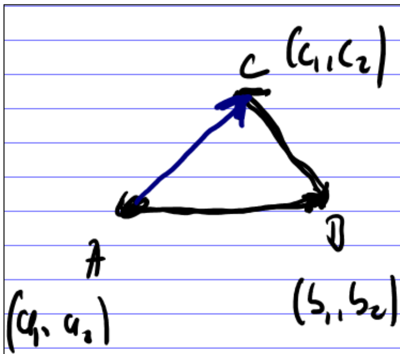
Panel 1



$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$+\vec{CA} = \vec{AC} + \vec{CA} = 0$$

$$\vec{AB} = \langle \cancel{b_1} - \cancel{a_1}, b_2 - a_2 \rangle$$

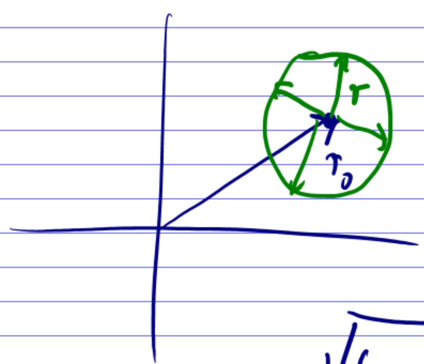
$$\vec{BC} = \langle \cancel{c_1} - \cancel{b_1}, c_2 - b_2 \rangle$$

$$\vec{CA} = \langle \cancel{a_1} - \cancel{c_1}, a_2 - c_2 \rangle$$

$$0 \qquad 0$$

1

Panel 2



$$\vec{r} = \langle x, y, z \rangle$$

$$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$$

$$\| r - r_0 \| = \| \langle x - x_0, y - y_0, z - z_0 \rangle \| = 1$$

$$\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = 1 \qquad \| \ \|^2$$

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = 1$$

sphere, radius 1

2

Panel 3

$a = 4i + j = \langle 4, 1 \rangle$

$b = i - 2j = \langle 1, -2 \rangle$

$a + b = \langle 5, -1 \rangle$

$4i + j + i - 2j = 5i - j$

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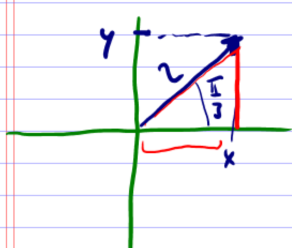
Panel 4

Other ways to describe vectors: Find a vector of length 2 that makes an angle of $\pi/3$ with positive x-axis.



$x = 2\cos(\pi/3)$

$y = 2\sin(\pi/3)$

Polar Coordinates

$\frac{x}{2} = \cos(\theta)$, $\frac{y}{2} = \sin(\theta)$

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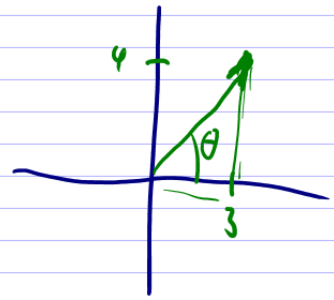
Panel 5

Other way around: find angle that $\vec{v} = 3i + 4j$ makes with the positive x-axis.



$$x = \|v\|\cos(\theta)$$

$$y = \|v\|\sin(\theta)$$

$$\Rightarrow \frac{y}{x} = \frac{\cancel{\|v\|}\sin(\theta)}{\cancel{\|v\|}\cos(\theta)} = \tan(\theta)$$

$$\tan(\theta) = \frac{4}{3} \quad \Rightarrow \quad \theta = \arctan\left(\frac{4}{3}\right)$$

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Panel 6

Know how to add/subtract vectors.

$$\vec{v} \cdot \vec{w} = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = \langle \cancel{v_1 \cdot w_1}, v_2 \cdot w_2 \rangle$$

$$\langle 0, 1 \rangle \cdot \langle 5, 0 \rangle = \vec{0} \quad \text{no good,}$$

because usually if $x \cdot y = 0 \Rightarrow x$ or y is zero

Def: Dot Product

$$\langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = v_1 w_1 + v_2 w_2$$

$$\langle v_1, v_2, v_3 \rangle \cdot \langle w_1, w_2, w_3 \rangle = v_1 w_1 + v_2 w_2 + v_3 w_3$$

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Panel 7

Examples of Dot Product

① $\langle 3,5\rangle \cdot \langle -1,2\rangle = (3\cdot -1) + (5\cdot 2) = -3+10 = \boxed{7}$

② $\langle 2,3\rangle \cdot \langle -3,2\rangle = (2\cdot -3)+(3\cdot 2) = -6+6 = \boxed{0}$

③ $\langle 1,-3,4\rangle \cdot \langle 1,5,2\rangle = (1\cdot 1) + (-3\cdot 5) + (4\cdot 2)$
$= 1 + -15 + 8 = -6$

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Panel 8

Properties of Dot Product

a) $\vec{a}\cdot\vec{a} = \|a\|^2$

b) $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$ distributive

c) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ commutative

$\vec{a} = \langle a_1, a_2\rangle$, $\vec{a}\cdot\vec{a} = \langle a_1,a_2\rangle\cdot\langle a_1,a_2\rangle =$
$= a_1^2 + a_2^2 = \|\langle a_1,a_2\rangle\|^2$ #

HW (for 2511 crowd)

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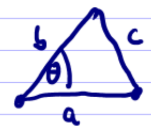
Panel 9

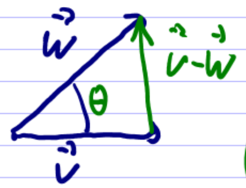
Theorem: If u and v are non-zero vectors in $\mathbb{R}^2$ then

$$\frac{u\cdot v}{\|u\|\cdot\|v\|} = \cos(\theta)$$

Proof



$$c^2 = a^2 + b^2 - 2ab\cos(\theta)$$



$$\|\vec v - \vec w\|^2 = \|v\|^2 + \|w\|^2 - 2\|\vec v\|\|\vec w\|\cos(\theta)$$

$$(\vec v - \vec w)\cdot(\vec v - \vec w) = \quad -\|-$$

$$\vec v\cdot\vec v - \vec v\vec w - \vec w\vec v + \vec w\cdot\vec w = \quad -\|-$$

$$\cancel{\|v\|^2} \cancel{-} 2\vec v\vec w + \cancel{\|w\|^2} = \cancel{\|w\|^2} + \cancel{\|v\|^2} \cancel{-} 2\|\vec v\|\|\vec w\|\cos(\theta)$$

$$\Rightarrow \frac{\vec v\vec w}{\|v\|\|w\|} = \cos\theta$$

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Panel 10

Ex: Find angle between $u = i - 2j + 2k$ = and

a) $v = -3i + 6j + 2k$

$$\frac{\vec v\cdot\vec w}{\|\vec v\|\cdot\|\vec w\|} = \cos(\theta)$$

$$\vec u\cdot\vec v = \langle 1,-2,2\rangle\cdot\langle -3,6,2\rangle$$
$$= -3 - 12 + 4 = -11$$

$$\|\vec u\| = \sqrt{9} = 3 \qquad \cos(\theta) = -11/21$$
$$\|\vec v\| = 7$$

b) $w = 2i + 7j + 6\vec k$

$$\vec u\cdot\vec w = \langle 1,-2,2\rangle\cdot\langle 2,7,6\rangle$$
$$2 + -14 + 12$$
$$0$$

Perpendicular

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Panel 11

Corollary: Two vectors $\vec{v}$ and $\vec{w}$ are perpendicular iff $\vec{v} \cdot \vec{w} = 0$

Ex: Which of the following vectors are perpendicular?

a) $\langle 1, 2, 3 \rangle$ and $\langle -1, -2, -3 \rangle$ $\quad -1 + -4 + -9$

b) $\langle 1, 2, 3 \rangle$ and $\langle -1, -3, 2 \rangle$ $\quad -1 + \cdot$

c) $\langle 1, 2, 3 \rangle$ and $\langle 6, -1, 1 \rangle$

d) $\langle 1, 2, 3 \rangle$ and $\langle 5, -1, 1 \rangle$

e) $\langle 1, 2, 3 \rangle$ and $\langle 0, -3, 2 \rangle$

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Panel 12

Ex: Find the angle that $\vec{a} = \langle 1, 2, 3 \rangle$ makes with the y-axis:

$$\cos(\theta) = \frac{\vec{a} \cdot j}{\|a\| \cdot \|j\|} = \frac{2}{\sqrt{14}}$$

$j = \langle 0, 1, 0 \rangle$

Directional Angles

$\vec{v}$ a vector. Then

angle with x-axis: $\cos(\theta) = v_1 / \|v\|$

angle with y-axis: $\cos(\theta) = v_2 / \|v\|$

angle with z-axis: $\cos(\theta) = v_3 / \|v\|$

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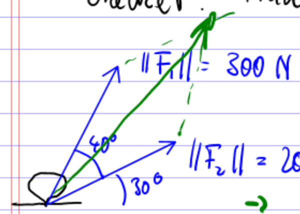
Panel 13

Application: Suppose two forces are applied to an eye bracket. Find the magnitude of the resultant force



$$\vec F_1 = \langle 300\cos(70), 300\sin(70)\rangle$$

$$\vec F_2 = \langle 200\cos(30), 200\sin(30)\rangle$$

$$\vec F_1 + \vec F_2 = \langle 275.8, 381.9\rangle$$

$$\|\vec F_1 + \vec F_2\| = 471$$ (units same)

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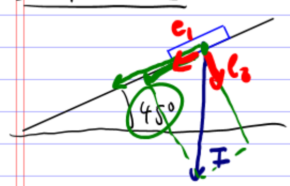
Panel 14

Application: Suppose a 10Kg block is on a 45° incline. What is the force pulling the block in the direction of the incline?



$$F = \langle 0, -10\rangle$$

$F = F_1 + F_2$, F_1, F_2 are perpendicular

$F = k_1\vec e_1 + k_2\vec e_2$ | $\vec e_1$, $\vec e_1, \vec e_2$ are perp. unit vector

$$\vec e_1 \cdot \vec F = k_1 \vec e_1 \cdot \vec e_1 + k_2 \vec e_2 \cdot \vec e_1 = k_1 + 0$$ want this!

$$k_1 = \frac{1}{\sqrt2}\langle -1,-1\rangle \cdot \langle 0,-10\rangle = \frac{10}{\sqrt2}$$

$$e_1 = \frac{1}{\sqrt2}\langle -1,-1\rangle$$

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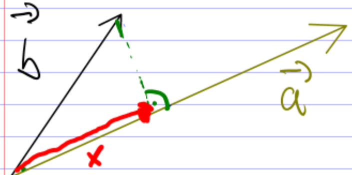
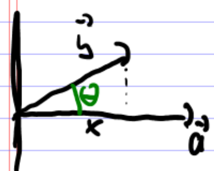
Panel 15

General Question: take two vectors $\vec{a}$ and $\vec{b}$. How much of $\vec{b}$ goes in the direction of $\vec{a}$?



or: what is the projection (shadow) of $\vec{b}$ onto $\vec{a}$



How long is x?

$$x = \|\vec{b}\| \cos(\theta) = \|\vec{b}\| \cdot \frac{\vec{a}\cdot\vec{b}}{\|a\|\,\|b\|} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|}$$

Length of projection of $\vec{b}$ onto $\vec{a}$: $\text{comp}_{\vec{a}}(\vec{b}) = \dfrac{\vec{a}\cdot\vec{b}}{\|a\|}$

Projection vector : $\text{proj}_{\vec{a}}(\vec{b}) = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|}\dfrac{a}{\|a\|} = \dfrac{\vec{a}\cdot\vec{b}}{\|a\|^2}\cdot\vec{a}$

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Panel 16

/

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Panel 17

Projection Formula: $\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \vec{a}$

Ex: Find length and direction of projection of $\vec{b} = \langle 1, 1, 2 \rangle$ onto $\vec{a} = \langle -2, 3, 1 \rangle$

$$\text{proj}_{\vec{a}}(\vec{b}) = \frac{\vec{a} \cdot \vec{b}}{\|a\|^2} \cdot \vec{a} = \left(\frac{3}{(\sqrt{14})^2}\right) \cdot \langle -2, 3, 1 \rangle$$

$$\text{comp}_{\vec{a}}(\vec{b}) = \|\text{proj}_{\vec{a}}(\vec{b})\| = \frac{3}{14} \cdot \sqrt{14} = 3/\sqrt{14}$$

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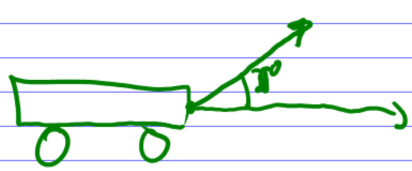
Panel 18

Application: A wagon is pulled a distance of 100 m by a constant force of 70 N, applied to a handle held at 35°. Find work done by F.



$W = \vec{F} \cdot \text{dist}$

$W = \text{proj}_{\vec{i}}(\vec{F}) \cdot 100$

rest in HW

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