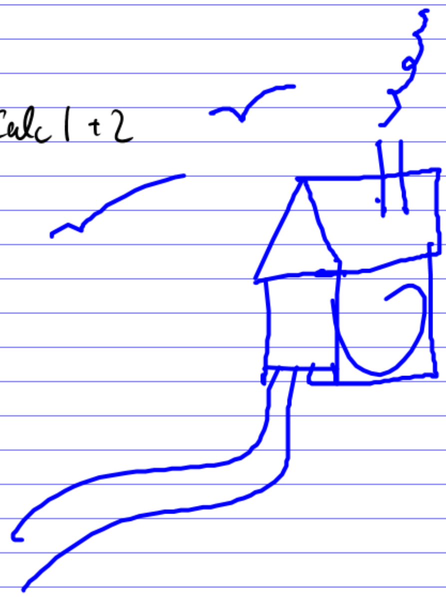
Panel 1

Last time

Review of Calc 1 + 2



Limit

Continuity

Differentiability

Integration

1

Panel 2

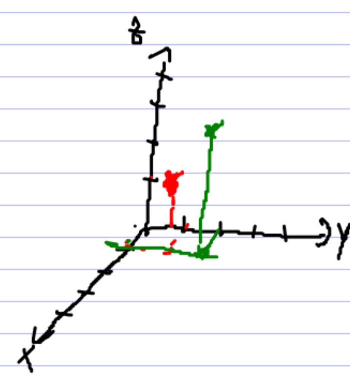
Introducing $\mathbb{R}^3$

Coordinate system in $\mathbb{R}^2$:

Ex: plot $(1,2)$, $(4,3)$

Coordinate system in $\mathbb{R}^3$:

Ex: plot $(1,1,1)$, $(1,2,3)$



x-axis: all points $(x, 0, 0)$

y-axis: $(0, y, 0)$

z-axis: $(0, 0, z)$

xy plane: all points $(x, y, 0)$

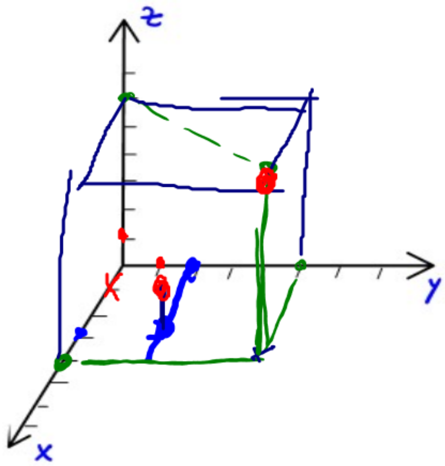
yz plane: $(0, y, z)$

xz plane: $(x, 0, z)$

2

Panel 3

Ex: Plot the following Points.



$P(3,2,1)$ - use blue

$Q(4,5,6)$ - use green

$I=(1,0,0)$

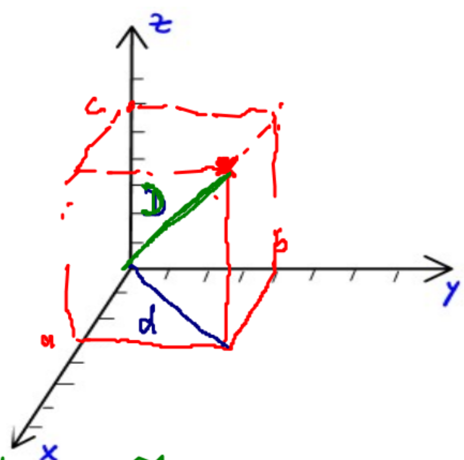
$J=(0,1,0)$

$K=(0,0,1)$

3

Panel 4

Distance in $\mathbb{R}^3$



$P(a,b,c)$

Distance between origin and P

$d=\sqrt{a^2+b^2}$

$D=\sqrt{d^2+c^2}=\sqrt{a^2+b^2+c^2}$

Distance between $P(x_0,y_0,z_0)$ and $Q(x_1,y_1,z_1)$:

$$D=\sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$$

4

Panel 5

$P(3,2,1)$ and $Q(4,5,6)$

Find distance to origin and distance P to Q

$$\sqrt{(4-3)^2+(5-2)^2+(6-1)^2}$$

$$\sqrt{1+9+25}$$

$$\sqrt{35}$$

5

Panel 6

3D Objects

$P(x,y,z) \Rightarrow d=\sqrt{x^2+y^2+z^2} \Rightarrow d^2=x^2+y^2+z^2$

is a sphere around $(0,0,0)$ with radius r.

Def: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=d^2$ is a sphere centered at (x_0,y_0,z_0) with radius of

Ex: $x^2+y^2+z^2-2x-4y+8z+17=0$

$$x^2-2x+1\underline{-1}+y^2-4y+4\underline{-4}+z^2+8z+16-16=-17$$

$$(x-1)^2+(y-2)^2+(z+4)^2=-17+1+4+16$$

$$=4$$

Center $(1,2,-4)$, radius 2.

6

Panel 7

Ex: Find the center + radius of the sphere

$$x^2 + y^2 + z^2 + 10x + 4y + 2z - 19 = 0$$

$$x^2 + 10x + 25 - 25 + y^2 + 4y + 4 - 4 + z^2 + 2z + 1 - 1 = 19$$

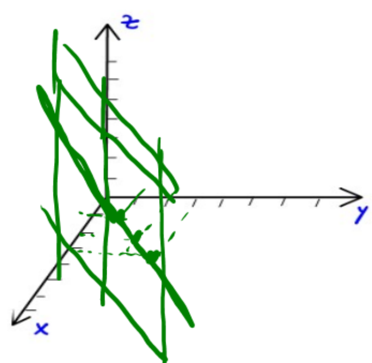
$$(x+5)^2 + (y+2)^2 + (z+1)^2 = 19 + 25 + 4 + 1 = 49$$

7

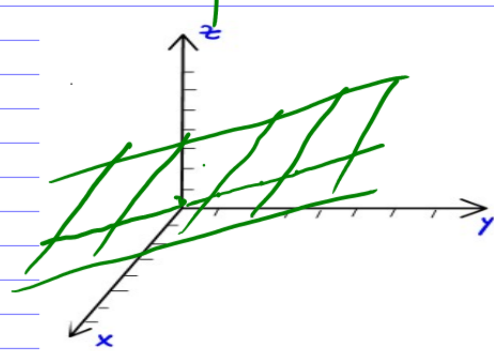
Panel 8

3D Objects

a) $y = x$

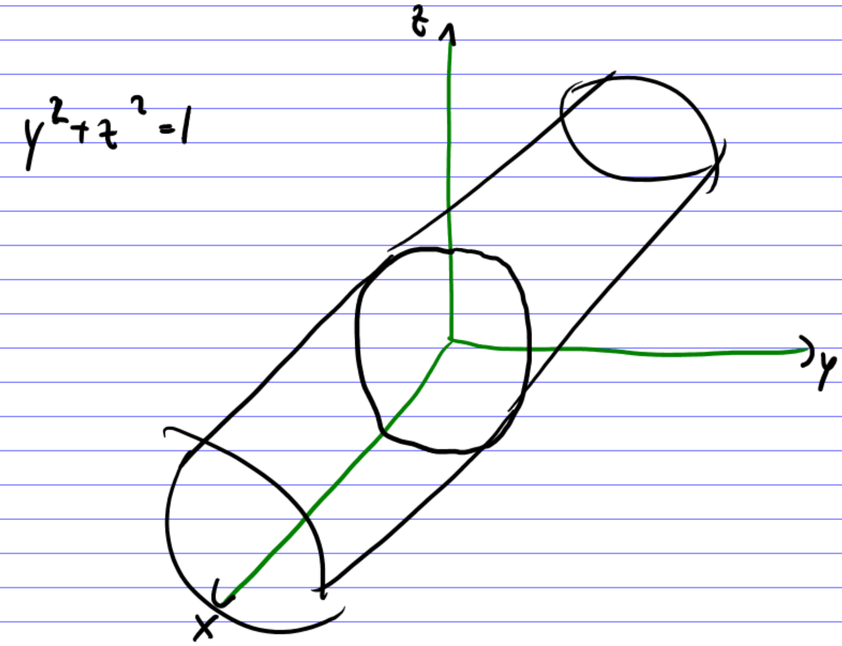


b) $z = \frac{1}{2} y$



8

Panel 9



9

Panel 10

Drawing 3D objects with Maple

Maple can easily draw 3D objects

Start Maple.

> $with(plots);$

> $implicitplot3d(z=y^2, x=-3..3, y=-3..3, z=-1..9);$

> $plot3d(x^2, x=-3..3, y=-3..3);$

> $implicitplot3d(z^2+y^2=4, x=-3..3, y=-3..3, z=-3..3);$

10

Panel 11

Ex: Use Maple to graph the following:
(use "Insert | Screen Grap" to paste into panel)

$x^2+y^2+z^2=4$ implicitplot3d

$y^2+z^2=2$ implicitplot3d

$(z=)\sin(x)\cdot\cos(y)$ plot3d

11

Panel 12

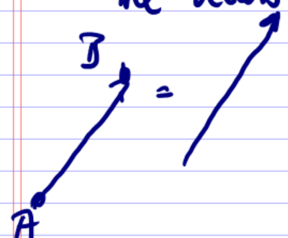
Vectors

Understand points in 3D (and 2D). Want to investigate more general objects $\Rightarrow$ vectors.

Def: A vector is a directed line segment, i.e. part of a line with a length and direction.

A vector $\vec{v}$ points from A to B is $\vec{v} = \overrightarrow{AB}$

All vectors with the same length and direction as $\overrightarrow{AB}$ are the same.

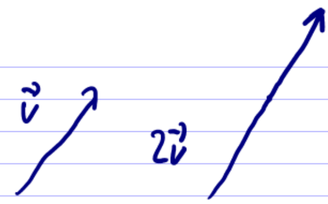


12

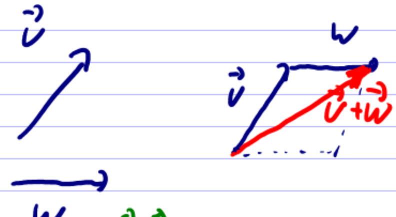
Panel 13

Vector Math, Geometrically

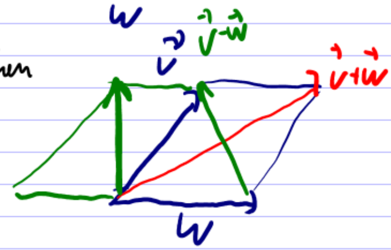
If $\vec{v}$ is a vector then
$k \cdot \vec{v}$ is $k \times$ length
of $\vec{v}$ (or opposite if $k<0$)



If $\vec{v}, \vec{w}$ are vectors, then
$\vec{v}+\vec{w}$ is vector s.t.
moving $\vec{w}$ to tip of $\vec{v}$



If $\vec{v}, \vec{w}$ are vectors, then
$\vec{v}-\vec{w}$ is $\vec{v}+(-\vec{w})$



Panel 14

Vector Math, Algebraically

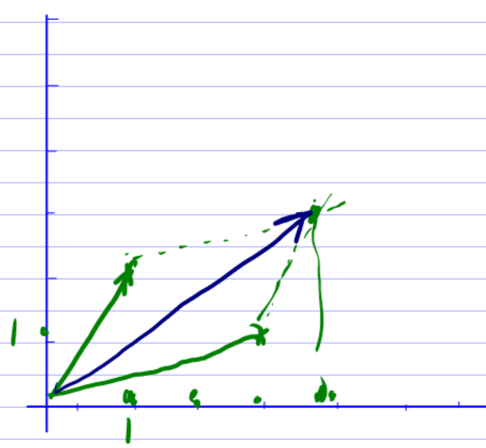
Algebraically v is described by components:
$\vec{v} = \langle v_1, v_2 \rangle$ or $\vec{v} = \langle v_1, v_2, v_3 \rangle$

Ex: Suppose $\vec{v} = \langle 1,2 \rangle$, $\vec{w} = \langle 3,1 \rangle$. Find

$\vec{v} + \vec{w}$ $\quad \langle 1,2\rangle + \langle 3,1\rangle =$
$\langle 4,3 \rangle$

$\vec{v} + 2\vec{w}$ $\quad \langle 1,2\rangle + 2\langle 3,1\rangle =$
$\langle 1,2\rangle + \langle 6,2\rangle =$
$\langle 7,4 \rangle$

$3\vec{v} - \vec{w}$ $\quad 3\langle 1,2\rangle - \langle 3,1\rangle$
$\langle 0,5 \rangle$



Panel 15

Vectors: Some Definitions

Def: The <u>Length</u> or <u>norm</u> of a vector $\vec{v} = (v_1, v_2)$ is:
$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}.$$

Def: A unit vector $\vec{v}$ is a vector such that
$$\|\vec{v}\| = 1$$

Note: If $\vec{v} = (v_1, v_2)$ is any non-zero vector, then
$$\vec{u} = \frac{1}{\|\vec{v}\|} \cdot \vec{v}$$
is a <u>unit</u> vector pointing in the same direction as $\vec{v}$.

15

Panel 16

Ex: $\langle \frac{1}{2}, \frac{3}{4} \rangle$ not unit vector

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$ is a unit vector

Ex: Find unit vector in the direction of $\vec{v} = (1, -5)$ and $\vec{w} = (3, 2, -1)$

$$\|\vec{w}\| = \sqrt{9+4+1} = \sqrt{14}$$

$$\vec{u} = \frac{1}{\sqrt{14}} \langle 3, 2, -1 \rangle$$

16