Panel 1

Abducted by an alien circus company, Professor Doyle is forced to write calculus equations in center ring.

# Math 2411 / 2511

## Calculus 3

Welcome!

Bert Wachsmuth

Panel 2

## About this Course:

Web Page http://pirate.shu.edu/~wachsmut

Syllabus username: 8-letters

HW ⇒ Dyknow / password: -same-

Maple

HW Alpha www.wolframalpha.com

Panel 3

## Calc I – Overview

| | Def. | Geometry | How-to |
|---|---|---|---|
| • Limit<br>$\lim_{x\to x_0} f(x)=L$ | given $\varepsilon>0$ $\exists\delta$<br>if $\lvert x-x_0\rvert<\delta$<br>$\Rightarrow \lvert f(x)-L\rvert<\varepsilon$ | as x gets close to $x_0$, what does f(x) get close to. | "plug in" - hope for the best<br>l'Hospital |
| • Continuity | $\lim_{x\to c} f(x)=f(c)$ | no gaps<br>no holes | polyn. are cont. |
| • Differentiation | $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ | slope of tangent | power rule<br>prod. rule<br>quot. rule |
| • Integration | $\int_a^b f(x)dx =$<br>$\lim$ Riemann sums | area under f<br>if $f\geq 0$ | anti derivative |

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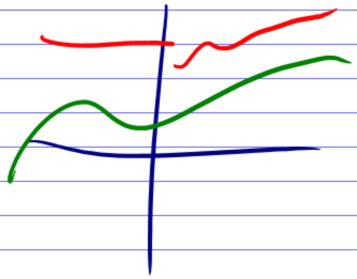
$$\lim_{x\to 5}\frac{x^2}{x-7}$$

$$\lim_{x\to 5}\frac{x^2-25}{x-5}=\lim_{x\to 5}\frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}}=\underline{10}$$



Panel 5

## Calc 1: Some Refresher

l'Hospital

① $\lim_{x \to 1} \frac{x^2-1}{x} = 0$ $\lim_{x \to 2} \frac{x^2-4}{x-2} = 4$ $\lim_{x \to 0} \frac{x \cdot \sin(x)}{1-\cos(x)}$

② $f(x) = x^2 - \sin(x) + e^x - \sqrt{x} + \ln(x)$

$$\lim_{x \to 0} \frac{\sin(x) + x\cos(x)}{\sin(x)} = \lim_{x \to 0} \frac{\cos(x) + \cos(x) - x\sin(x)}{\cos(x)} = \frac{2}{1} = 2$$

$$g(x) = \frac{x \sin(x)}{\sqrt{1-x^2}} = \frac{(\sin(x) + x\cos(x))\sqrt{1-x^2} - x\sin(x)\left[\frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x)\right]}{1-x^2}$$

③ $\int 5x - 3\sqrt{x} + e^x - \frac{5}{x} - \frac{6}{x^2}\, dx$

$$5\frac{1}{2}x^2 - 3\frac{2}{3}x^{3/2} + e^x - 5\ln(x) - 6(-1)x^{-1} + C$$

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## Calc 2 - Overview

- Heavy Duty Integration Techniques
  e.g. trig. subst, int by parts, PFD
- Applications of Integration
  e.g.: center of mass, rotational solids
- Sequences & Series
  e.g: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- Differential Equations
  e.g. $y' = ky$

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## Calc 2 Refresher

$$\cancel{\int}(uv)' = \int u'v + \int uv'$$

$$\underline{uv} = \int u'v + \boxed{\int uv'}$$

$$\int uv' = \underline{uv} - \int u'v$$

① $\int \underset{u}{x} \cdot \underset{v'}{\sin(x)}\, dx$

$$= -x\cos(x) + \int 1 \cos(x)\, dx \checkmark$$

② $\int \frac{5x}{x^2 - x - 2}\, dx = \frac{10}{3} \int \frac{1}{x-2}\, dx + \frac{5}{3} \int \frac{1}{x+1}\, dx = \frac{10}{3}\ln|x-2| + \frac{5}{3}\ln|x+1|$

$$\frac{5x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} = \frac{A(x+1) + B(x-2)}{(x-2)(x+1)}$$

$$5x = A(x+1) + B(x-2)$$

$x = 2$: $10 = 3A \Rightarrow A = \frac{10}{3}$

$x = -1$: $-5 = -3B \Rightarrow B = 5/3$

③ $\frac{5x}{1+x^2}$ as a power series

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$$\frac{5x}{1+x^2} = 5x \cdot \boxed{\frac{1}{1-(-x^2)}} = 5x \sum_{n=0}^{\infty} (-x^2)^n$$

$$= 5x\left(1 - x^2 + x^4 - x^6 + x^8 - x^{10} \dots\right)$$

$$= \underline{5x - 5x^3 + 5x^5 - 5x^7 + 5x^9 - \dots}$$

$$1 + x + x^2 + x^3 + \dots + x^N = S$$

$$- \quad x + x^2 + x^3 + x^4 + \dots + x^{N+1} = xS$$

$$1 - x^{N+1} = S(1-x)$$

$$\lim \frac{1 - \boxed{x^{N+1}}}{1-x} = \boxed{S}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

$x = 1/2$: $\lim_{N \to \infty} \left(\frac{1}{2}\right)^{N+1} = 0$

Panel 9

## Calc 3

$f(x,y) = x^2 + y^2$

Calc 3 = Calc 1 + Calc 2 + multiple variables =

⇒ Introducing $\mathbb{R}^3$

(vectors, def, norm, dot + cross product)

⇒ Vector-valued functions

(def., limit, cont., diff, normal vector, curvature, motion)

⇒ Functions of two variables

(def., limit, cont., partial deriv, gradients, max/min)

⇒ Multiple Integrals

(double/triple int., area, volume, surface area, centroids)

⇒ Icing on cake

(Vector fields, Green's thm, div. theorem, Stoke's thm)

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$f: \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^2$

$f: \mathbb{R}^{\overset{3}{2}} \to \mathbb{R}$ $\quad f(x,y) = x^2 + y^2$

$f: \mathbb{R} \to \mathbb{R}^{\overset{3}{2}}$ $\quad f(t) = \langle \cos(t), \sin(t) \rangle$

$f: \mathbb{R}^{\overset{3}{2}} \to \mathbb{R}^{\overset{3}{2}}$ $\quad f(x,y) = \langle xy, x^2 + y^2 \rangle$

$f: \mathbb{R}^5 \to \mathbb{R}$ $\quad f(x_1, x_2, x_3, x_4, x_5) = x_1 + \overset{2}{x_2} + \overset{3}{x_3} + \overset{2}{x_4} + x_5$