Panel 1

Monday - Stoke's thm (new)

Wed + Fri : Review

Friday: get Final Take home

due the day/time of our final exam (last day of finals)

Extra credit: Monday

1

Panel 2

Last Time:

C a curve defined by $y = f(x)$, $x \in [a,b]$. Then

$$\int_C g(x,y)\, ds = \int_a^b g(x, f(x)) \sqrt{1+(f')^2}\, dx$$

S a surface defined by $z = f(x,y)$, $(x,y) \in D \subset \mathbb{R}^2$. Then

$$\iint_S g(x,y,z)\, dS = \iint_D g(x,y,f(x,y)) \underbrace{\sqrt{1+f_x^2+f_y^2}}_{\text{grad to } z-f=0}\, dA$$

S as above with normal vector $\vec{n} = \langle -f_x, -f_y, 1\rangle \frac{1}{\sqrt{f_x^2+f_y^2+1}}$.

$\vec{F}$ a 3D vector field. Then

Flux

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iint \langle M, N, P\rangle \cdot \langle -f_x, -f_y, 1\rangle \cancel{\frac{1}{\sqrt{1+f_x^2+f_y^2}}}\, \cancel{\sqrt{1+f_x^2+f_y^2}}\, dA =$$

$$= \iint_D -Mf_x - Nf_y + 1\, dA$$

2

Panel 3

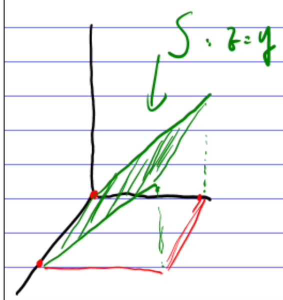
Ex: Let S be the surface defined by $z = y$ where $(x,y) \in [0,1] \times [0,1]$. Find

$$\iint_S x^2 + y^2 + z^2 \, dS$$

$f_x = 0$

$z = f(x,y) = y \Rightarrow f_y = 1 \Rightarrow$

$\Rightarrow dS = \sqrt{1 + f_x^2 + f_y^2} \, dA = \sqrt{2} \, dA$



$$\iint (x^2 + y^2 + (y)^2) \sqrt{2} \, dA = \sqrt{2} \iint x^2 + 2y^2 \, dA =$$

$$= \sqrt{2} \int_0^1 \int_0^1 x^2 + 2y^2 \, dx \, dy$$

$= \boxed{\text{Maple}}$

3

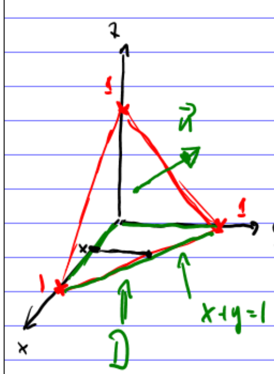
Panel 4

Ex: Let S be the surface defined by $x + y + z = 1$ split by the coordinate planes and let $\vec{F}$ be the vector field $\vec{F} = \langle \overset{M}{x}, \overset{N}{xy}, \overset{P}{xyz} \rangle$. Find the flux of $\vec{F}$ through S.

surface defined by $x + y + z = 1$

i.e. $z = \underline{1 - x - y}$ ✓

$x + y + z = 1$



$\vec{n} = \langle -f_x, -f_y, 1 \rangle \cdot \frac{1}{\|\ \|}$ $\quad f_x = -1, \; f_y = -1$

$= \langle 1, 1, 1 \rangle \frac{1}{\sqrt{3}}$

$$\iint_S \vec{F} \cdot \vec{n} \, dS = \iint_D x + xy + xy(1 - x - y) \, dA$$

$$= \int_0^1 \int_0^{1-x} x + yx + xy(1 - x - y) \, dy \, dx$$

$=$ Maple

4

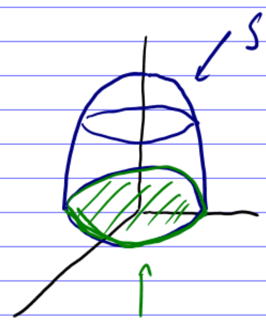
Panel 5

Ex: Let S be $z = 9 - x^2 - y^2$, $z \geq 0$ and $\vec{F} = \langle 3x, 3y, z \rangle$.
Find flux of $\vec{F}$ through S.

$z = f(x,y) = 9 - x^2 - y^2$

$f_x = -2x$

$f_y = -2y$



$\iint_S \vec{F} \cdot \vec{n}\, dS = \iint_D 6x^2 + 6y^2 + (9 - x^2 - y^2)\, dA =$

$= \iint_D 9 + 5x^2 + 5y^2\, dA$

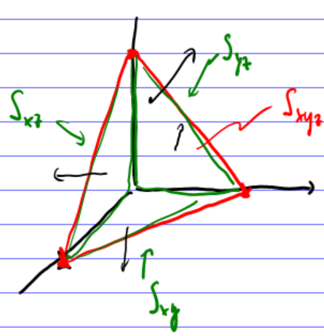
$z = 0$:
$9 - x^2 - y^2 = 0$
$9 = x^2 + y^2$

D circle radius 3

Polars $\int_0^{2\pi}\int_0^3 (9 + 5r^2) \cdot r\,dr\,d\theta =$ Maple

Panel 6

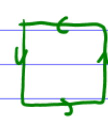
Ex: Let S be the closed surface bounded by $x + y + z = 1$ and by the coordinate planes. Let $\vec{F}$ be the vector field $\vec{F} = \langle x, xy, xyz \rangle$, as before. Find flux of $\vec{F}$ through the closed surface S:



$\iint_{S_{xyz}} \vec{F} \cdot \vec{n}\, dS + \iint_{S_{yz}} + \iint_{S_{xz}} + \iint_{S_{xy}}$

too much!

Recall: $\oint_C F\,dr$



Panel 7

The Divergence Theorem:

Let Q be a region in $\mathbb{R}^3$ bounded by a closed surface S with outward normal $\vec{n}$. If $\vec{F}$ is a 3D vector field with cont. derivatives, then

$$\oiint_S \vec{F}\cdot\vec{n}\, dS = \iiint_Q \operatorname{div}(\vec{F})\, dV$$

"Flux of $\vec{F}$ over S is a tripple integral of divergence"

7

Panel 8

Compare Green's Thm and Divergence Thm:

Green: $\vec{F}$ 2D vector field, C closed curve

$$\Rightarrow \oint_C \vec{F}\, d\vec{r} = \iint_D \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$$

$$= \iint_D \operatorname{curl}(\vec{F})\cdot\vec{k}\, dA$$

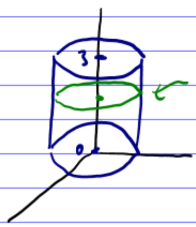
Divergence: $\vec{F}$ 3D vector field, S closed surface

$$\oiint_S \vec{F}\cdot\vec{n}\, dS = \iiint_Q \operatorname{div}(\vec{F})\, dV$$

8

Panel 9

Ex: Let S be the surface defined by $x^2+y^2=4$, $z=0$, and $z=3$. If $\vec{F}=\langle x^3, y^3, z\rangle$ find flux of $\vec{F}$ through S.



$$\iint_S \vec{F}\cdot\vec{n}\,dS = 3 \text{ integrals or}$$

$$= \iiint \operatorname{div}(\vec{F})\,dV =$$

$$= \iiint 3x^2+3y^2+1\,dV$$

$$= \int_0^3\int_0^{2\pi}\int_0^2 (3x^2+3y^2+1)\,r\,dr\,d\theta\,dz$$

$$= \int_0^3\int_0^{2\pi}\int_0^2 (3r^2+1)\,r\,dr\,d\theta\,dz = \boxed{\text{Maple}}$$

9

Panel 10

Tripple Integrals

$$\int_a^b f(x)\,dx \overset{\text{def}}{=} \lim_{n\to\infty}\sum_{i=1}^n f(x_i)\,\Delta x$$

height, width

$$\int_a^b\int_c^d f(x,y)\,dA \overset{\text{def}}{=} \lim_{n\to\infty}\sum_{i,j=1}^n f(x_i,y_j)\,\Delta A$$

height, area

$$\iiint f(x,y,z)\,dV \overset{\text{def}}{=} \lim_{n\to\infty}\sum_{i,j,k=1}^n f(x_i,y_j,z_k)\,\Delta V$$

height, volume

$$= \iiint_{\text{bounds}} f(x,y,z)\,dx\,dy\,dz$$

$dy\,dx\,dz$

$dz\,dx\,dy$

$\vdots$

hard

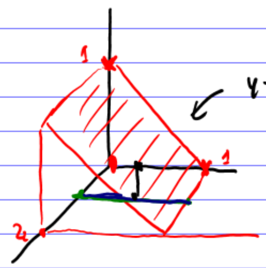
10

Panel 11

Ex: Q is the region bounded by xz-plane, xy-plane, the planes x = 0 and x = 2 as well as y + z = 1

Find
$$\iiint_Q xy \, dV = \int_0^2 \int_0^1 \int_0^{1-y} xy \, dz \, dy \, dx$$

$\boxed{\text{Maple}}$

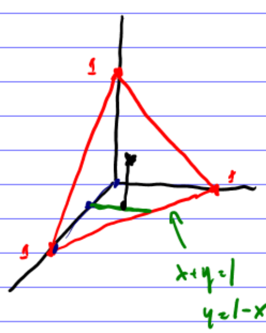


11

Panel 12

Ex: Q is the region bounded by the three coordinate planes and by x + y + z = 1. Find
$$\iiint_Q z \, dV$$

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} z \, dz \, dy \, dx = \underline{\text{Maple}}$$



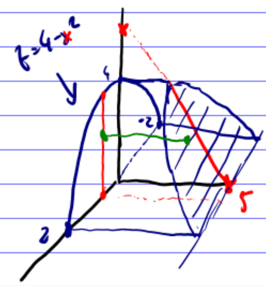
12

Panel 13

Ex: Q region bdd by $z = 4 - x^2$, $y + z = 5$, xy and xz-planes. Find

$$\iiint_Q 4x^2 \, dV = \int_{-2}^{2} \int_{0}^{4x^2} \int_{0}^{5-z} 4x^2 \, dy \, dz \, dx =$$

Maple



13

Panel 14

Ex: Q region bdd by $z = 4 - x^2$, $y + z = 5$, xy and xz-planes.

Let $\vec{F} = \langle x^3 + \sin(z), x^2 y + \cos(z), e^{x^2+y^2} \rangle$, find $\iint_R \vec{F} \cdot \vec{n} \, dS$

HW

14