Panel 1

Last Time

Function $f(x,y)$ — $\int f\,ds$, $\int f\,dx$, $\int f\,dy$

Vector field $F = \langle M, N \rangle$ – $\int_C \vec{F}\,d\vec{r} = \int_C M\,dx + N\,dy \;(+ P\,dz)$

Fundamental Thm. of Line Integration:

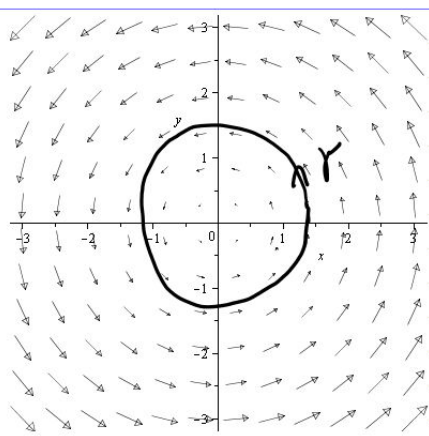
$$\int_C \vec{F}\,d\vec{r} = f(B) - f(A) \quad \text{if}$$

F is conservative with $\nabla f = F$, f potential

1

Panel 2

$F(x,y) = \langle -y, x \rangle$, $\gamma = \langle r\cos(t), r\sin(t) \rangle$, $t \in [0, 2\pi]$. Then



$\int_C \vec{F}\,d\vec{r}$ pos or neg.

$$\int F\,d\vec{r} = \int_C M\,dx + N\,dy$$

$$= \int_C -y\,dx + x\,dy =$$

$$= +\int_0^{2\pi} r^2\sin^2(t) + r^2\cos^2(t)\,dt = r^2\, 2\pi$$

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Panel 3

Fundamental Theorem for Line Integrals

If F is conservative with potential function f, and $\gamma(t)$, $a \le t \le b$, a smooth curve. Then:

$$\int_\gamma \vec{F}\, d\vec{r} = f(\gamma(b)) - f(\gamma(a))$$

Consequences: If F is conservative then:

(1) $\int_{\gamma_1} F\, d\vec{r} = \int_{\gamma_2} F\, dr$ for all γ_1, γ_2 having same start and end (path independence)

(2) $\oint_C F\, dr = 0$, for all closed loops C

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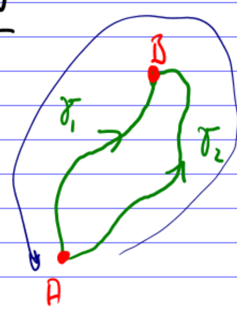
Panel 4

Conservative Vector Fields

$$\int_{\gamma_1} \vec{F}\, d\vec{r} = f(B) - f(A)$$

$$\int_{\gamma_2} \vec{F}\, d\vec{r} = f(B) - f(A)$$

$$\oint_\gamma \vec{F}\, d\vec{r} = f(A) - f(A) = 0$$



Thm: A vector field $\vec{F} = \langle M, N \rangle$ defined in an open, simply connected region with M, N having cont. partials. Then

$$\oint \vec{F}\, d\vec{r} = 0 \quad \forall \text{ closed curves } C \iff F \text{ conserv.}$$

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Panel 5

Find a conservative vector field that has the given potential:

$f(z,y,z) = \sin(x^2 + y^2 + z^2)$

on Monday

Find $div(\nabla \cdot F)$ and $curl(F) = \nabla \times F$

$F(x,y,z) = < x^2 z, y^2 x, y + 2z >$

Evaluate $\int_C (x-y)dx + xdy$ if C is the graph of $y^2 = x$ from (4,-2) to (4,2)

Find the work done by F(x,y,z) along the curve $< t, t^2, t^3 >$ from (0, 0, 0) to (2, 4, 8), where $F(x,y,z) = < y,z,x >$

Check which of the following vector fields is not conservative.

$F(x,y) = < 3x^2 y + 2, x^3 + 4y^3 >$

$F(x,y) = < e^x, 3 - e^x \sin(y) >$

$F(x,y,z) = < 8xz, 1 - 6yz^2, 4x^2 - 9y^2 z^z >$

Show that the line integrals are independent of the path, and find their value:

$$\int_{(-1,2)}^{(3,11)} (y^2 + 2xy)dx + (x^2 + 2xy)dx$$

$$\int_{(1,0,2)}^{(-2,1,3)} (6xy^3 + 2z^2)dx + (9x^2y^2)dy + (4xz + 1)dz$$

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Panel 6

Ex: Let $F(x,y) = \left(\frac{y^2}{1+x^2}, 2y \arctan(x) \right)$ and

$r(t) = (t^2, 2t)$, $t \in [0, 1]$. Find $\int_\Gamma \vec{F}\, d\vec{r}$

(1) $\int \vec{F}\, d\vec{r} = \int M dx + N dy = \int_C \frac{y^2}{1+x^2} dx + 2y \arctan(x)\, dy =$ antideriv of arctan

$= \int_0^1 \frac{4t^2}{1+t^4} 2t\,dt + 4t \arctan(t^2)\, dt$

(2) $\frac{\partial M}{\partial y} = \frac{2y}{1+x^2} = \frac{\partial N}{\partial x} \Rightarrow$ conservative, so I need $f(x,y) = y^2 \arctan(x)$ guessing

$\Rightarrow \int \vec{F}\, d\vec{r} = y^2 \arctan(x) \Big|_{(0,0)}^{(1,2)} = 4 \arctan(1) - 0 = 4 \arctan(1) =$

$= 4 \cdot \pi/4 = \pi$

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Panel 7

Ex: Find $\int_\gamma \overset{M}{\tan(y)}\,dx + \overset{N}{x\sec^2(y)}\,dy = 0$.

where $\gamma(t) = \langle \cos(t), \sin(t) \rangle$, $t \in [0, 2\pi]$

$\frac{\partial M}{\partial y} = \sec^2(y)$, $\frac{\partial N}{\partial x} = \sec^2(y)$ $\Rightarrow$ conservative

γ is closed curve

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Panel 8

$\int \cancel{Mdx}, \cancel{Ndy}$

$\int \vec{F}\,d\vec{r}$ important because it gives Work

$\int_\gamma \vec{F}\,d\vec{r}$ → long way; → short-cut (conservative) — γ not closed

$\oint_\gamma \vec{F}\,d\vec{r}$ — long way; — (short cut) conservative 0; — short cut — γ closed

Green's Thm $\Rightarrow$ next!

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