Panel 1

Definitions: Please state in your own words the following definitions:

a) Limit of a function $z = f(x, y)$

b) Continuity of a function $z = f(x, y)$

c) partial derivative of a function f(x,y)

d) gradient and its properties

e) directional derivative of a function f(x, y) in the direction of a unit vector u

~~f) total differential~~

g) The (definition and geometric meaning of) the double integral of f over the region R $\iint_R f(x, y)dA$

h) Surface area

surface plots + contour plots

1

Panel 2

Theorems: Describe, in your own words, the following:

a) ~~a theorem relating differentiability with continuity~~

b) a theorem stating criteria for a function to have relative extrema

c) a result that classifies critical points into relative max., min., or saddle points

d) the procedure to find relative extrema of a function f(x, y)

e) the procedure to find absolute extrema of a function f(x, y)

f) how to switch a double integral to polar coordinates

g) a theorem that allows you to evaluate a double integral easily

h) the "change of variables" theorem to change from rectangular to polar coordinates

diff'ble → cont.

cont → diff'ble

2

Panel 3

True/False questions:

a) If $\lim_{(x,y)\to(0,0)} f(x,y) = 0$ then $\lim_{x\to 0} f(x,0) = 0$ T $\lim_{x\to 0} f(x,0) = 0 \Rightarrow \lim_{(x,y)\to(0,0)} f(x,y) = 0$

b) If $\lim_{y\to 0} f(0,y) = 0$ then $\lim_{(x,y)\to(0,0)} f(x,y) = 0$ F

c) $\lim_{h\to 0} \frac{f(x+ah,y+bh)-f(x,y)}{h} = \frac{\partial}{\partial x} f(x,y)$ F

d) If f is continuous at (0,0), and f(0,0) = 10, then $\lim_{(x,y)\to(0,0)} f(x,y) = 10$ T

e) ~~If f(x, y) is continuous, it must be differentiable~~

f) ~~If f(x, y) is differentiable, it must be continuous~~

g) If f(x, y) is a function such that all second order partials exist and are continuous then $f_{xy} = f_{yx}$

h) The volume under f(x,y), where $a \le x \le b$ and $g(x) \le y \le h(x)$ is $\int_a^b \int_{g(x)}^{\cancel{g(x)}\ h(x)} f(x,y)\,dydx$ y x

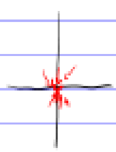
i) If $f(x,y)$ is continuous then $\int_a^b \int_c^d f(x,y)\,dydx = \int_c^d \int_a^b f(x,y)\,dxdy$ T

3

Panel 4

#5)

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$



$x=0$: $\lim_{\substack{y\to 0\\ x=0}} \frac{0-y^2}{0+y^2} = -1$

$y=0$: $\lim_{x\to 0} \frac{x^2-0}{x^2+0} = 1$

$\Big\}\ \lim_{(x,y)\to(0,0)} (\quad)$ d.n.e.

If $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

$\Rightarrow \lim_{x\to 0} f(x,0) = 0 = \lim_{y\to 0} f(0) = 0$

4

Panel 5

$$\frac{\partial}{\partial x} f(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$\lim_{h \to 0} \frac{f(x+ah, y+bh) - f(x,y)}{h} = D_{\vec{u}}(f), \quad \vec{u} = \langle a,b \rangle$$

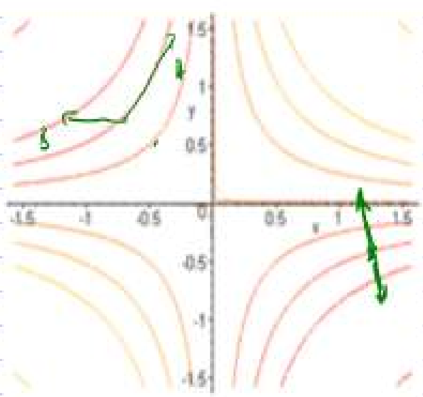
f is cont. at (0,0) : $\underset{(x,y)\to(0,0)}{\lim} \overset{(2)}{f(x,y)} \overset{(3)}{=} \overset{(1)}{f(0,0)}$ $= 10$

4) Domain of $f(x,y) = \frac{1}{x^2+y^2}$ $\iff x^2+y^2 = 0 \iff (x,y) = 0$

Domain is $\mathbb{R}^2 - \{(0,0)\}$

5

Panel 6



Gradient: (f_x, f_y)

points in dir. of largest incr.

$\to \|\nabla f\|$ is largest increase

perp. to level curves

$f(x,y) = x^2 e^y$. max. value of dir. deriv. at $(-2,0)$

a) $D_u(f)$, $u = \langle 1,1 \rangle$: $\nabla f \cdot \frac{1}{\sqrt{2}}(1,1)$, $\nabla f = \langle \overset{-4}{2xe^y}, \overset{4}{x^2 e^y} \rangle$

$\|\nabla f\| = \sqrt{(2xe^y)^2 + (x^2 e^y)^2} = \sqrt{32}$

6

Panel 7

$f(x,y) = 3x^2 - 2xy + y^2 - 8y$

(1) $f_x = 6x - 2y = 0 \qquad 12 - 2y = 0$

$f_y = -2x + 2y - 8 = 0$

$4x - 8 = 0$, $x = 2, y = 6$ critical point

(2) $H = \begin{pmatrix} 6 & -2 \\ -2 & 2 \end{pmatrix}$, $D = 12 - 4 = 8$

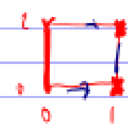
(3) $D > 0$, $f_{xx} = 6 > 0 \Rightarrow$ min at $(2,6)$

7

Panel 8

$f(x,y) = 3x^2 - 2xy + y^2 - 8y$ abs. extrema over $[0,1] \times [0,2]$

(1) Critical point: $\cancel{(2,6)}$ ✓



(2) $x = 0$: $f(y) = y^2 - 8y$, $f'(y) = 2y - 8 = 0$, $\cancel{y=4}$ ✓

$x = 1$: $f(y) = 3 - 2y + y^2 - 8y = y^2 - 10y + 3$, $f'(y) = 2y - 10 = 0$, $\cancel{y=5}$

$y = 0$: $f(x) = 3x^2 \Rightarrow x = 0$

$y = 2$: $f(x) = 3x^2 - 4x + 4 - 16 = 3x^2 - 4x - 12$, $f'(x) = 6x - 4 = 0$,

$x = \frac{4}{6}$

	$f(x,y)$
$(0,0)$	0
$(\frac{2}{3}, 2)$	
$(1,0)$	
$(1,2)$	
$(0,2)$	

pick largest

→ smallest

8

Panel 9

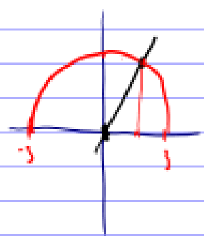
$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\,dy\,dx = \int_{0}^{\pi}\int_{0}^{3} r \cdot r\,dr\,d\theta =$$

$$= \int_{0}^{\pi}\int_{0}^{3} r^2\,dr\,d\theta \quad \text{easy}$$

$x = r\cos\theta$

$y = r\sin\theta$

$dy\,dx = r\,dr\,d\theta$



$y = 0$ to $y = \sqrt{9-x^2}$ or $x^2+y^2=9$

9

Panel 10

$$\int_{0}^{a}\int_{0}^{b}\int_{0}^{c} x^2+y^2+z^2\,dx\,dy\,dz = \int_{0}^{a}\int_{0}^{b}\left[\tfrac{1}{3}x^3 + xy^2 + xz^2\Big|_{0}^{c}\right]dy\,dz =$$

$$= \int_{0}^{a}\int_{0}^{b} \tfrac{1}{3}c^3 + cy^2 + cz^2\,dy\,dz$$

$$= \int_{0}^{a} \tfrac{1}{3}c^3 y + \tfrac{1}{3}cy^3 + cz^2 y\Big|_{y=0}^{y=b} dz =$$

$$= \int_{0}^{a} \tfrac{1}{3}c^3 b + \tfrac{1}{3}cb^3 + cz^2 b\,dz =$$

$$= \tfrac{1}{3}c^3 bz + \tfrac{1}{3}cb^3 z + \tfrac{1}{3}cz^3 b\Big|_{0}^{a} =$$

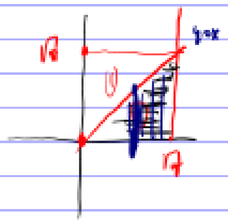
$$= \boxed{\tfrac{1}{3}c^3 ba + \tfrac{1}{3}b^3 ca + \tfrac{1}{3}a^3 cb}$$

10

Panel 11

f) $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\,dx\,dy = \int_0^{\sqrt{\pi}} \int_0^{x} \cos(x^2)\,dy\,dx = \int_0^{\sqrt{\pi}} y\cos(x^2)\Big|_{y=0}^{y=x} dx =$

x = y, and x = $\sqrt{\pi}$



$= \int_0^{\sqrt{\pi}} x\cos(x^2)\,dx =$

$= \frac{1}{2}\sin(x^2)\Big|_0^{\sqrt{\pi}} =$

$= \frac{1}{2}\left(\sin(\pi) - \sin(0)\right) = 0$

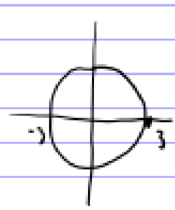
11

Panel 12

$\text{area}(f) = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\,dA = \iint \sqrt{4x^2 + 4y^2 + 1}\,dA =$

$f(x,y) = z = 16 - x^2 - y^2$ above $x^2 + y^2 = 9$

$f_x = -2x,\ f_y = -2y \Rightarrow \sqrt{f_x^2 + f_y^2 + 1} = \sqrt{4x^2 + 4y^2 + 1}$



$= \int_0^{2\pi}\int_0^{3} \sqrt{4r^2 + 1}\, r\,dr\,d\theta$

$\boxed{\text{subst.}}$

$\boxed{\text{Proofs}}$ (c) (f) (g)

12

Panel 13

13