

Panel 1


Least Time: Integration in \mathbb{R}^2

$$\iint_D f(x,y) dA = \lim_{n \rightarrow \infty} \sum_{i=1}^m \sum_{j=1}^n f(x_i, y_j) \Delta x_i \Delta y_j \quad \textcircled{1} \text{ Definition}$$

Volume under $f(x,y)$ over D , if $f > 0$ ② Geometry

$$\iint_a^b \int_c^d f(x,y) dx dy = \iint_c^d \int_a^b f(x,y) dy dx \quad \textcircled{3} \text{ How-to}$$

Fubini



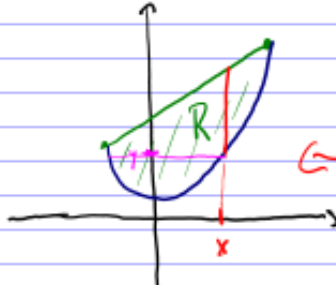
$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y) dy dx$$

$$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y) dx dy$$

(Next: Online work sheet Int, diff)

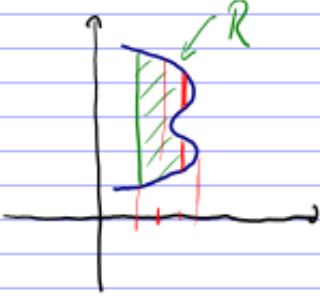
Panel 2

$dx dy$ or $dy dx$?



① $\iint_R f(x,y) dx dy$ y-fix

② $\iint_R f(x,y) dy dx$ x-fix



① $\iint_R f(x,y) dx dy$ →

② $\iint_R f(x,y) dy dx$

Panel 3

Which picture represents $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y) dy dx$

The left graph shows a region bounded by a curve and the x-axis. A vertical red line segment is drawn at a point x on the x-axis, extending from the x-axis to the curve. The region is labeled with $g_1(x)$ and $g_2(x)$ in red. The x-axis has tick marks at a and b . A circled letter 'A' is to the left of the graph.

The right graph shows a region bounded by two curves. A horizontal red line segment is drawn at a point y on the y-axis, extending from the left curve to the right curve. The region is labeled with $g_1(y)$ and $g_2(y)$ in red. The y-axis has tick marks at a and b . A circled letter 'B' is to the right of the graph.

3

Panel 4

Name: _____

Quiz #7

① Classify the relative extrema, if any, for the function $f(x,y) = x^2 - 8x + y^4 - 4y$

4

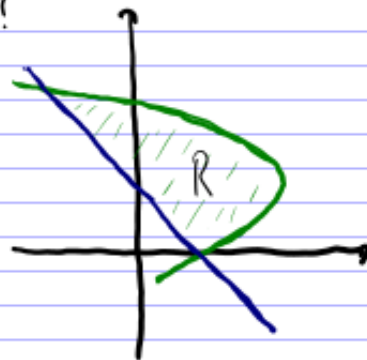
Panel 5

② Compute $\int_0^2 \int_0^1 6xy^2 dx dy$

③ Which integration order is easier?

a) $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y) dy dx$

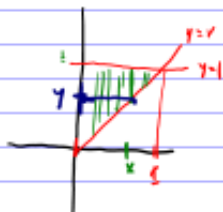
b) $\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y) dx dy$



Panel 6

Occasionally there are other reasons that you would prefer one integration order over another - after all, you do have choices.

Ex: Find $\int_0^1 \int_0^1 \sin(y^2) dy dx$ ^{no anti-derivative} $= \int_0^1 \int_0^1 \sin(y^2) dx dy$



$y=x$ to $y=1$ $= \int_0^1 [x \sin(y^2)]_{x=0}^1 dy =$

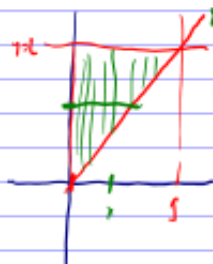
$= \int_0^1 y \sin(y^2) dy =$

$= \frac{1}{2} \cos(y^2) \Big|_0^1 = \frac{1}{2} \cos(1) + \frac{1}{2} \cos(0)$

Panel 7

$$\int_0^1 \int_0^y \sqrt{1-y^2} dx dy = \int_0^1 x \sqrt{1-y^2} \Big|_0^y dx = \int_0^1 y \sqrt{1-y^2} dy = \dots$$

$$= -\frac{1}{3} (1-y^2)^{3/2} \Big|_0^1$$

$$\int_0^1 \int_{2x}^2 e^{y^2} dy dx = \int_0^2 \int_0^{z/2} e^{y^2} dx dy \quad (\text{HW})$$


$y = 2x$ to $y = 2$

7