Panel 1

Sheets + Coordinate Systems

$x^2 + z^2 = 1$ tube around y-axis

Vectors: add, subtract, length, visually
dot prod., cross product, angles
projection

Planes + Lines: parametric or scalar equations, intersections, distances, angles

Vector-valued functions: limits, deriv, integrals
tangents, unit tangent, normal, binormal, length,
curvature

Motion in Space: velocity, speed, accel., normal + tangential
comp. of accel., story problem

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Panel 2

Formulas:

angle between $\vec{v}$ and $\vec{w}$: $\cos(\theta) = \dfrac{\vec{v}\cdot\vec{w}}{\|v\|\cdot\|w\|}$

$\mathrm{proj}_{\vec{v}}(\vec{w}) = \dfrac{\vec{v}\cdot\vec{w}}{\|v\|^2}\,\vec{v}$

$r(t) = \langle\ \rangle$: $T = \dfrac{r'(t)}{\|r'(t)\|}$

$N = \dfrac{T'(t)}{\|T'(t)\|}$

$B = T \times N$

$k = \chi = \dfrac{\|T'\|}{\|r'\|} = \dfrac{\|r' \times r''\|}{\|r'\|^3}$

$s = \int_a^b \|r'(t)\|\,dt$

$\|a\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$v = r'(t)$

$s = \|r'(t)\|$

$a = r''(t)$

$a_T = \dfrac{v \cdot a}{s}$

$a_N = \dfrac{\|v \times a\|}{s}$

Supporting plane: spanned by T and N

Osculating circle: circle in supp. plane with radius χ

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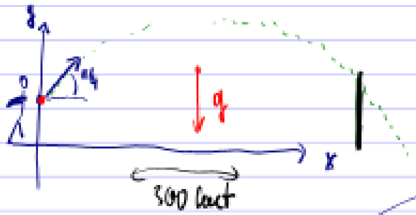
Panel 3

Application of Motion

A baseball is hit 3 feet above ground at 100 feet per second and at an angle of Pi/4 with respect to the ground. Find the maximum height reached by the baseball. Will it clear a 10-foot high fence located 300 feet from home base?



$r(t) =$

Know: $r(0) = \langle 0, 3 \rangle$

$v(0) = \frac{100}{\sqrt{2}} \langle 1, 1 \rangle$

$a(t) = \langle 0, -g \rangle$

$a(t) = \langle 0, -g \rangle$

$v(t) = \langle c_1, -gt + c_2 \rangle = \langle \frac{100}{\sqrt{2}}, -gt + \frac{100}{\sqrt{2}} \rangle$

$r(t) = \langle \frac{100}{\sqrt{2}} t + d_1, -\frac{1}{2} g t^2 + \frac{100}{\sqrt{2}} t + d_2 \rangle = \langle \frac{100}{\sqrt{2}} t, -\frac{1}{2} g t^2 + \frac{100}{\sqrt{2}} t + 3 \rangle$

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Panel 4

$$a(t) = \langle \overbrace{\frac{100}{\sqrt{2}} t}^{x(t)}, \overbrace{-\frac{1}{2} g t^2 + \frac{100}{\sqrt{2}} t + 3}^{y(t)} \rangle$$

① Max height. $y(t) = -\frac{1}{2} g t^2 + \frac{100}{\sqrt{2}} t + 3$ max. $y'(t) = 0 \Rightarrow t_0 = \ldots$

$y(t_0)$ is max. height

② Does it clear a 10-foot wall at 300 feet?

Want: $x = \frac{100}{\sqrt{2}} t = 300 \Rightarrow t = 300 \cdot \frac{\sqrt{2}}{100} = 3\sqrt{2}$

Height: $y(3\sqrt{2}) = -\frac{1}{2} 32 \cdot (3\sqrt{2})^2 + \frac{100}{\sqrt{2}} 3\sqrt{2} + 3 = 15$

$= -16 \cdot 18 + 300 + 3$

HR!

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Panel 5

True / False questions

$r(t) = \langle t^3, 2t^3, 3t^3 \rangle$ is a line $\sim \langle u, 2u, 3u \rangle$ line YES ($t^3 = u$)

$\frac{d}{dt}\left(v(t) \times w(t)\right) = v'(t) \times w'(t)$ No – try an example

$\frac{d}{dt}\|r(t)\| = \|r'(t)\|$ No try example

If $\|r(t)\| = 5$ for all t then $r(t)$ is constant F

If $\|r(t)\| = 5$ for all t then $r(t) \cdot r'(t) = 0$ T

More about distances, planes, and vectors

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Panel 6

Vectors: Suppose $u = \langle 7, -2, 3 \rangle$, $v = \langle -1, 4, 5 \rangle$, and $w = \langle -2, 1, -3 \rangle$

1. Are u and v orthogonal, (yes) parallel, or neither?
   Find graphically and algebraically $2u + 3v$ and $u - v$
   Find the angle between v and w
   Find $u \cdot v$ (dot product), $u \times v$ (cross product), $u \cdot (v \times w)$, and $\|u\|$ ✓

$7 \cdot -1 + -2 \cdot 4 + 3 \cdot 5 = -7 - 8 + 15 = 0$

two vectors are parallel if $c\vec{w} = \vec{v}$

$c\vec{v} = \langle -1, 4, 5 \rangle = \langle -2, 1, -3 \rangle$ No!

$v \cdot (v \times w) = w \cdot (v \times w) = 0$

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Panel 7

Planes / Lines:

param. Equation of line through $P(1,2,3)$ and $Q(4,5,1)$

$$\ell(t) = \langle 1,2,3\rangle + t\langle 3,3,-2\rangle = P + t\vec{PQ}$$

scalar Equation of plane through $P(1,0,2)$, $Q(2,1,3)$, $R(1,1,1)$

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0 \quad \text{where } \langle a,b,c\rangle = \vec{PQ}\times\vec{PR}$$

$$ax+by+cz+\underline{d}=0$$

~~Intersection of lines / planes~~

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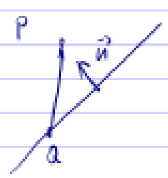
Panel 8

Distances

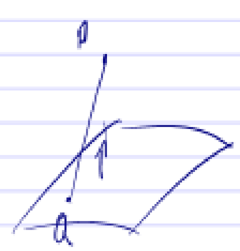
Find distance of $\underline{P}(3,1)$ to line $2x - y = 1$ ($ax+by=1$)



$$d = \frac{\vec{PQ}\cdot n}{\|n\|}, \quad n = \langle 2,-1\rangle, \quad Q=\langle 0,-1\rangle$$

$$Q = \langle 1,1\rangle$$

Find distance of $P(1,0,3)$ to plane $2x+3y+z=0$



same as above.

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Panel 9

If $r(t) = < 4t, t^2, t^3 >$, find $r'(t)$, $r''(t)$, $\frac{d}{dt}\|r(t)\|$

If $r(t) = < e^t, 3t^3, \frac{3}{6t} >$ some curve, find $\int_1^2 r(t)dt$

If $r(t) = < t, \frac{1}{t} >$, find $T(t)$, $N(t)$, a_t and a_n

HW If $r(t) = \langle 3+t, 2t, 1-4t \rangle$, find N. Explain.

$r(t) = \langle t, \frac{1}{t} \rangle$

$r'(t) = \langle 1, -\frac{1}{t^2} \rangle$, $\|r'\| = \sqrt{1 + (1/t^2)^2} = \sqrt{1 + 1/t^4}$

$T = \frac{1}{\sqrt{1 + 1/t^4}} \langle 1, -1/t^2 \rangle \Rightarrow N = \frac{1}{\sqrt{1/t^4 + 1}} \langle \frac{1}{t^2}, 1 \rangle$

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Panel 10

If $r(t) = \langle 3-3t, 4t, t \rangle$ find length of curve as $t \in [0,1]$

$s = \int_0^1 \|r'(t)\| dt = \int_0^1 \|\langle -3, 4, 1 \rangle\| dt = \int_0^1 \sqrt{26}\, dt = \sqrt{26}$

be prepared for int/parts and subst.

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Panel 11

Find curvature of $r(t) = \langle t, 3t^2, \frac{t^2}{2} \rangle$ at $t = 1$

$$\kappa = \frac{\|T'\|}{\|r'\|} = \frac{\|r' \times r''\|}{\|r'\|^3}$$

$r'(t) = \langle 1, 6t, t \rangle \Rightarrow r'(1) = \langle 1, 6, 1 \rangle$

$r''(t) = \langle 0, 6, 1 \rangle \Rightarrow r''(1) = \langle 0, 6, 1 \rangle$

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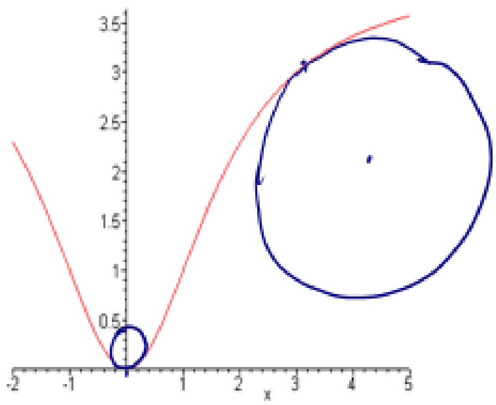
Panel 12

Picture Problems:

7. Picture: Sketch the circle that fits the graph below the best at the points x = 0 and x = 3. At which of the two points is the curvature smaller?



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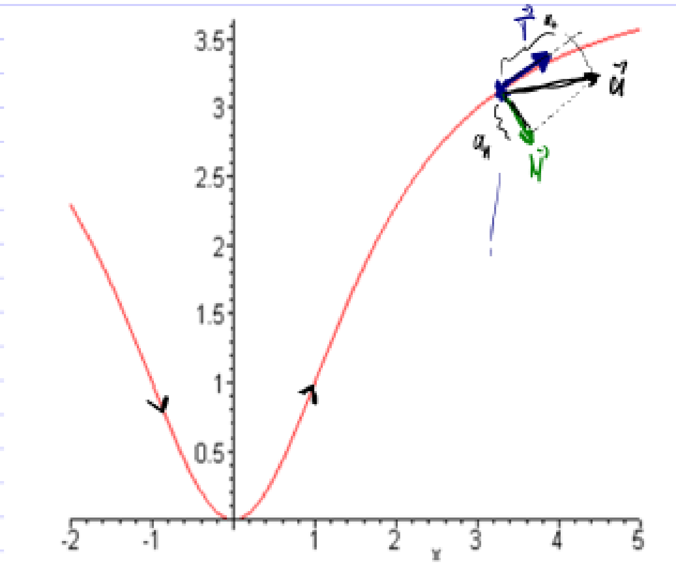
Panel 13

Picture Problems:

Sketch $\vec{T}$, $\vec{N}$, $\vec{a}$, a_T, and a_N at $t = 3$



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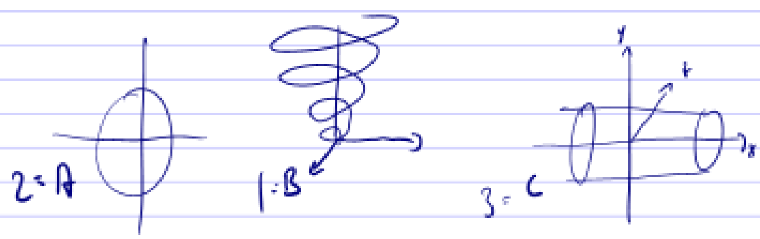
Panel 14

Picture Problems

Match graphs to functions

slinkys, spirals, tubes, etc.



① $r(t) = \langle t\sin(t), t\cos(t), t \rangle$

② $g(t) = \langle \cos(t), \sin(t) \rangle$ ③ $y^2 + z^2 = 1$

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Panel 15

Story Problem:

For sure!

What is the maximum height and range of a projectile fired at a height of 3 feet above the ground with an initial velocity of 900 feet/sec and at an angle of 45 degrees above the horizontal?

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Panel 16

Proofs:

Prove the following facts:

1. Show that $u \times v = -(v \times u)$
2. Show that $u \cdot (v \times u) = 0$
3. Show that if $y = f(x)$ is a function that is twice continuously differentiable, then the curvature of f at a point x is $K = \frac{|f''(x)|}{\left(1+[f'(x)]^2\right)^{3/2}}$
4. Prove that the curvature of a line in space is zero.

$y = f(x) \Rightarrow r(t) = \langle t, f(t), 0 \rangle$

$x \quad y$

$$\frac{\|r' \times r''\|}{\|r'\|^3}$$

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