Panel 1

Chapter 12 Review

$\mathbb{R}^3$: points, spheres, and sheets

Vectors: add, subtract, length, unit vector

Dot product: used to find angle between vectors

Cross product: used to find a vector perp. to two vectors in $\mathbb{R}^3$

Lines: vector and scalar equations

Planes: vector and scalar equations

Distances: between points, lines, planes, ...

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Panel 2

Name: ______________

Quiz #3

(1) Find equation of a line through $P(1,1,2)$ and $Q(2,3,4)$.

(x) vector: $P_0 + t\vec{v}$

scalar: $ax + by + c = 0$

(2) Find equation of plane through $P(1,1,2)$, $Q(2,3,4)$, and $R(3,2,1)$.

vector: $P_0 + t\vec{v} + s\vec{w}$

(x) scalar: $ax + by + cz + d = 0$

$\langle a,b,c \rangle$ is normal to plane.

$$v = \vec{PQ} = \qquad w = \vec{PR} = \qquad \begin{array}{ccc} i & j & k \end{array}$$
$$\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -1 \end{vmatrix} =$$

$$\langle -2-2, -(-1-4), 1-4 \rangle = \langle -4, 5, -3 \rangle$$

$$-4x + 5y - 3z + d = 0$$

$$-4\cdot 1 + 5\cdot 1 - 3\cdot 2 + d = 0 \Rightarrow \underline{d = 5}$$

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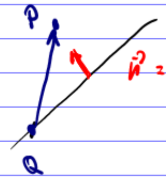
Panel 3

let $x=0 \Rightarrow y=$

③ Find distance of $P(1,-2)$ to line $2x-y=0$

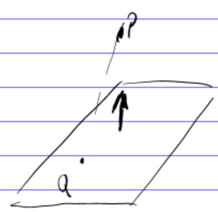


$Q(0,0)$ or $Q(7,14)$

$\vec{n} = \langle 2,-1 \rangle$

$\vec{PQ} = \langle -1,2 \rangle \qquad \| \mathrm{proj}_{\vec{n}} \vec{PQ} \| = \frac{|\langle -1,2\rangle \cdot \langle 2,-1\rangle|}{\sqrt{5}} = 4/\sqrt{5}$

④ Find distance of $P(1,-2,3)$ to $x+y+z=0$



$\neq 0$

Bonus: Represent the difference of scores in Superbowl 2010 as an integral. Hint: Saints 31 – Colts 17

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Panel 4

Super Bowl 2010:

Difference in scores:

$$\int_{17}^{31} 1\,dx$$

$$28 \int_0^\infty x e^{-x^2}\,dx$$

$$28 \int_0^{1-e^{-2}} \sum_{n=0}^{\infty} x^n\,dx$$

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Panel 5

Space Curves: $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle = \langle f(t), g(t), h(t) \rangle$

$\vec{r} : \mathbb{R} \to \mathbb{R}^3$

Examples: $r_1(t) = \langle 2t, -1-t, 2+t \rangle$ - line

$r_2(t) = \langle t, \sin(t), \cos(t) \rangle$ - slinky around x

$r_3(t) = \langle t \cdot \sin(t), t \cdot \cos(t), t \rangle$ spiral around z

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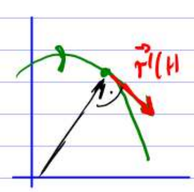
Panel 6

Derivatives of Space Curves aka Vector-valued functions

If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ and f, g, h are differentiable

then $\vec{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$



Ex: $\vec{r}(t) = \langle 1+t^3, te^{-t}, \sin(2t) \rangle$

Find $\vec{r}(0)$ and $\vec{r}'(0)$

Compute $\vec{r}(0) \cdot \vec{r}'(0)$

$r(t) = \langle 1+t^3, te^{-t}, \sin(2t) \rangle$, $\vec{r}(0) = \langle 1, 0, 0 \rangle$

$r'(t) = \langle 3t^2, e^{-t} - te^{-t}, 2\cos(2t) \rangle$, $\vec{r}'(0) = \langle 0, 1, 2 \rangle$

$\vec{r}(0) \cdot \vec{r}'(0) = 0$

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Panel 7

Ex. Find equation of tangent line to $r(t) = \langle 2\cos t, \sin t, t \rangle$ at the point $P(0, 1, \pi/2)$

$\ell(t) = P_0 + t\vec{v}$

$P_0\ (0, 1, \pi/2)$

$\vec{v} = \vec{r}\,'(t)\Big|_{t=\pi/2} = \langle -2\sin(t), \cos(t), 1 \rangle\Big|_{t=\pi/2} = \langle -2, 0, 1 \rangle$

$\ell(t) = (0, 1, \pi/2) + t\langle -2, 0, 1 \rangle =$

$= \langle -2t, 1, \pi/2 + t \rangle$

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Panel 8

Proof: $\frac{d}{dt}\left[\vec{u}(t) \cdot \vec{v}(t)\right] = \vec{u}\,'(t) \cdot \vec{v}(t) + \vec{u}(t) \cdot \vec{v}\,'(t)$

$u(t) = \langle u_1(t), u_2(t) \rangle, \quad v(t) = \langle v_1(t), v_2(t) \rangle.$

$\Rightarrow \frac{d}{dt}\, u \cdot v = \frac{d}{dt}\, u_1(t)v_1(t) + u_2(t)v_2(t) + u_3(t)v_3(t) =$

$= u_1'v_1 + u_1v_1' + u_2'v_2 + u_2v_2' + u_3'v_3 + u_3v_3'$

Laptop crashed!

Rest as HW

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