Panel 1

## Last Time

**Lines:**

$ax + by + c = 0$ 2D scalar equation

$l(t) = P_0 + t\vec{v}$ 2D, 3D, 4D, 5D, ... vector equation

$\langle a,b \rangle$ perp. to $\vec{v}$ in 2D

**Planes:**

$ax + by + cz + d = 0$ 3D scalar equation

$p(t,s) = P_0 + t\vec{v} + s\vec{w}$ 3D, 4D, 5D vector equation

$\langle a,b,c \rangle$ perp. to all vectors in the plane

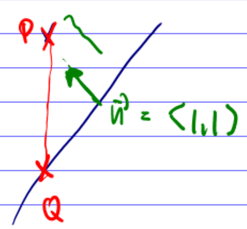
perp to $\vec{v}, \vec{w}$

1

Panel 2

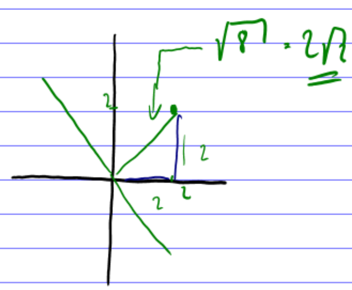
Ex: Find distance of $P(2,2)$ from line $x + y = 0$

$Q(1,-1)$




$$d = \frac{|\vec{PQ} \cdot \vec{n}|}{\|n\|} = \frac{|\langle -1,-3 \rangle \cdot \langle 1,1 \rangle|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

True pic

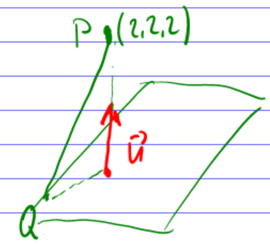



Important: $\text{proj}_{\vec{v}}(\vec{w}) = \frac{v \cdot w}{\|\vec{v}\|^2} \cdot \vec{v}$ , $\|\text{proj}\| = \frac{|v \cdot w|}{\|v\|}$

2

Panel 3

Ex: Find distance of $\overset{P}{(2,2,2)}$ from $x+y+z=0$




$\vec{n} = \langle 1,1,1 \rangle$

$Q = \langle 0,0,0 \rangle \Rightarrow PQ = \langle -2,-2,-2 \rangle$

$$d = \frac{|PQ \cdot \vec{n}|}{\|\vec{n}\|} = \frac{|\langle -2,-2,-2 \rangle \langle 1,1,1 \rangle|}{\sqrt{3}} = \frac{6}{\sqrt{3}}$$

3

Panel 4

Distance Formulas between:

$P_0(x_0,y_0) \in \mathbb{R}^2$ and line $ax+by+c=0$ in $\mathbb{R}^2$

$$d = \frac{|ax_0+by_0|}{\sqrt{a^2+b^2}}$$ don't want to memorize

$P_0(x_0,y_0,z_0) \in \mathbb{R}^3$ and plane $ax+by+cz+d=0$ in $\mathbb{R}^3$

$$d = \frac{|ax_0+by_0+cz_0|}{\sqrt{a^2+b^2+c^2}}$$ but do know the recipe!

Distance between 2 lines

4

Panel 5

Distance between $\ell_1(t) = \langle t, 2t, 3t \rangle$
and $\ell_2(t) = \langle 1-t, t, t \rangle$

$v_1 = \langle 1, 2, 3 \rangle$, $v_2 = \langle -1, 1, 1 \rangle$ not parallel, so they could intersect.

If they did: $\ell_1(t) = \ell_2(s)$

$t = 1 - s$ ← no check!

$$\left| \begin{array}{l} 2t = s \\ 3t = s \end{array} \right. \Rightarrow 2t = 3t \Rightarrow t = 0,\ s = 0$$

Thus, they do not intersect!

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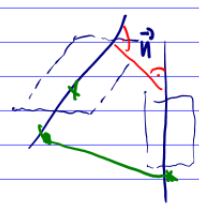
Panel 6

Distance between $\ell_1(t) = \langle t, 2t, 3t \rangle$
and $\ell_2(t) = \langle 1-t, t, t \rangle$

They don't intersect. The distance occurs at a vector perp. to both lines!




$$\vec{u} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -1 & 1 & 1 \end{vmatrix} = \langle 2-3, -(1+3), 1+2 \rangle = \langle -1, -4, 3 \rangle$$

Any point on $\ell_1: (0) = (0,0,0) = P$
on $\ell_2(0) = (1, 0, 0) = Q$

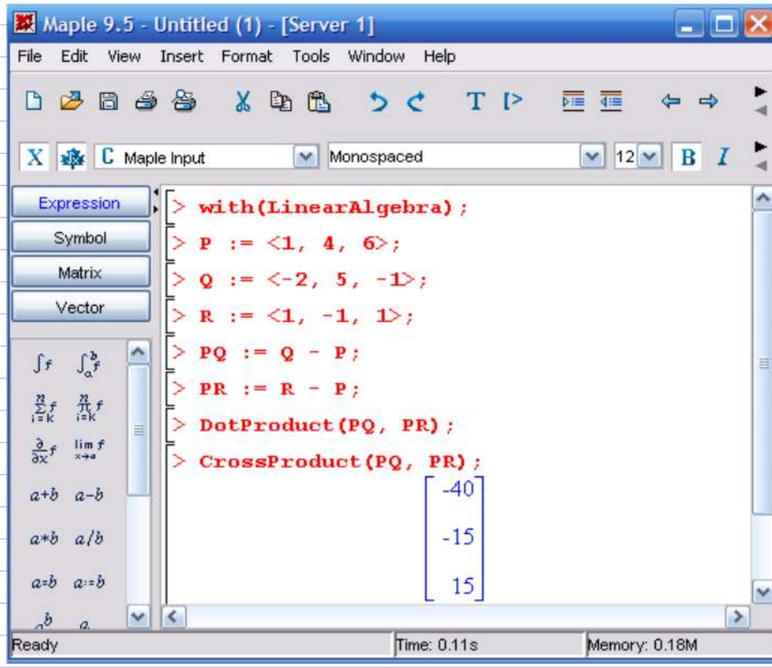
dist: $\dfrac{\vec{PQ} \cdot \vec{u}}{\|u\|} = \dfrac{\langle -1,0,0 \rangle \langle -1,-4,3 \rangle}{\sqrt{26}}$

$= \underline{1/\sqrt{26}}$

6

Panel 7

## Maple: Dot + Cross Product

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Panel 8

## Pop Quiz

Use Maple or compute manually:

(1) $\langle 1,3,9 \rangle \cdot \langle 2,-2,-5 \rangle$

a) 0 c) -44

b) 44 d) 12

(2) $\langle 1,3,9 \rangle \times \langle 2,-2,-5 \rangle$

a) 0 b) $\langle 3,-68,-23 \rangle$

c) $\langle 1,2,3 \rangle$ d) $\langle 3,68,-23 \rangle$

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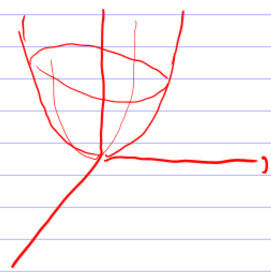
Panel 9

Section 12.6 : Quadratic Surfaces

skip for now

$z = x^2 + y^2$

$f(x, y) = x^2 + y^2 = z$



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Panel 10

Chapter 12 Review

started with $\mathbb{R}^3$, points, spheres, and sheets $y = x^2$

vectors : add, subtracted, mult by scalar, length.

Dot product: angles, perp., projection

Cross product: perp to 2 vectors, area of parallelogram

Lines

Planes

Distances

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Panel 11

Space curves: $r: \mathbb{R} \to \mathbb{R}^3$, t is time

Def: $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ is a vector-valued function with component functions f, g, and h

Many concepts work as they should: If $\vec{r}(t)$ is vector-valued function then

Limit: $\lim_{t \to t_0} \vec{r}(t) = \langle \lim_{t \to t_0} f(t), \lim_{t \to t_0} g(t), \lim_{t \to t_0} h(t) \rangle$

Derivative: $\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$

Integral: $\int \vec{r}(t)\,dt = \langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \rangle$

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Panel 12

Ex: $r(t) = \left\langle \frac{\sin(t)}{t}, \frac{\cos(t)-1}{t}, \frac{e^{t^2}-1}{t^2} \right\rangle$

$$\lim_{t \to 0} r(t) = \left\langle \lim_{t \to 0} \frac{\sin(t)}{t} = \lim_{t \to 0} \frac{\cos(t)}{1} = 1 \right.$$

$$\lim_{t \to 0} \frac{\cos(t)-1}{t} = \lim \frac{-\sin(t)}{1} = 0$$

$$\left. \lim_{t \to 0} \frac{e^{t^2}-1}{t^2} = \lim_{t \to 0} \frac{2te^{t^2}}{2t} = 1 \right\rangle$$

$$= \langle 1, 0, 1 \rangle$$

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Panel 13

The problem with vector-valued functions is to visualize them, and interprate the deriv. + integrals:

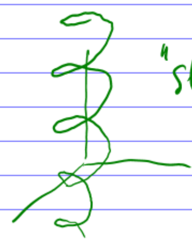
Ex: $\vec{r}(t) = \langle 1+t, 2+5t, -1+6t \rangle$ - describe graph

$= (1,2,-1) + t\langle 1,5,6 \rangle$

line

Ex: $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$ - describe graph

space curves



"slinky"

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Panel 14

Sketch graph of

$r_1(t) = \langle (4+\sin(20t))\cos(t), (4+\sin(20t))\sin(t), \cos(20t) \rangle$

$r_2(t) = \langle (2+\cos(1.5t))\cos t, (2+\cos(1.5t))\sin(t) \quad \sin(1.5t) \rangle$

Maple:

with (plots)

plot3d - surface

implicit plot3d - surface

spacecurve( [x(t), y(t), z(t)], t=a..b)

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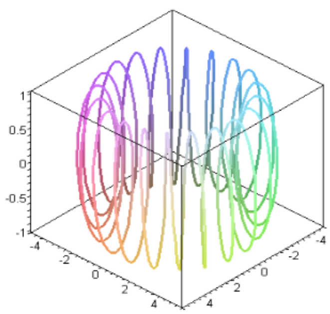
Panel 15

Sketch graph of

$r_1(t) = \langle (4+\sin(20t))\cos(t), (4+\sin(20t))\sin(t), \cos(20t) \rangle$

$r_2(t) = \langle (2+\cos(1.5t))\cos t, (2+\cos(1.5t))\sin(t)\ \ \sin(1.5t) \rangle$

```
> with(plots);
> spacecurve( [(4+sin(20*t))*cos(t), (4+sin(20*t))*sin(t), cos(20*t)], t=0..2*Pi, numpoints=500);
> spacecurve( [(2+cos(1.5*t))*cos(t), (2+cos(1.5*t))*sin(t), sin(1.5*t)], t=0..4*Pi, numpoints=500);
> spacecurve([t, t^2, t^3], t=0..2);
```




try

tubeplot

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