Panel 1

Last time:

Maple to draw 3D surfaces and sheets

Vectors: directed line segments — geom

add - subtr - mult. by scalar — alg.

Length of a vector $\vec{v} = \langle v_1, v_2, v_3 \rangle$: $\|v\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

Unit vector $\vec{u}$: $\|\vec{u}\| = 1$

Vector from $P(x_1, y_1)$ to $Q(x_2, y_2)$: $\vec{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

Dot Product: $\langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = v_1 w_1 + v_2 w_2$

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Panel 2

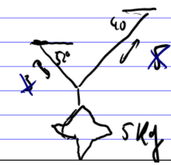
Ex: $\vec{v} = \langle 3, 4, -2 \rangle$

$\|\vec{v}\| = \sqrt{3^2 + 4^2 + (-2)^2} = \sqrt{29}$

Unit vector $\vec{u}$ in the direction of $\vec{v}$: $\vec{u} = \frac{1}{\sqrt{29}} \vec{v}$

$\Rightarrow \vec{u} = \left\langle \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \right\rangle$.

$\|u\| = \sqrt{\frac{9}{29} + \frac{16}{29} + \frac{4}{29}} = \sqrt{\frac{29}{29}} = 1$



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Panel 3

Properties of Dot Product

a) $\vec{a} \cdot \vec{a} = \|a\|^2$

b) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

c) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

(a) Proof $\vec{a} = \langle a_1, a_2, a_3 \rangle$; $\vec{a} \cdot \vec{a} = \langle a_1, a_2, a_3 \rangle \cdot \langle a_1, a_2, a_3 \rangle =$

$a_1 a_1 + a_2 a_2 + a_3 a_3 = a_1^2 + a_2^2 + a_3^2 = \|a\|^2$

(b) Proof HW for the 25th gang

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Panel 4

Ex: Find angle between $u = i - 2j + 2k = \langle 1, -2, 2 \rangle$ and

a) $v = -3i + 6j + 2k = \langle -3, 6, 2 \rangle$

$\cos(\theta) = \frac{u \cdot v}{\|u\| \|v\|}$; $u \cdot v = \langle 1, -2, 2 \rangle \cdot \langle -3, 6, 2 \rangle = -3 - 12 + 4 = -11$

$\|u\| = \sqrt{9} = 3$, $\|v\| = \sqrt{49} = 7$ | $\cos\theta = -\frac{11}{3 \cdot 7} = -\frac{11}{21}$

$\theta = \text{inv}\cos(-11/21)$

$= \arccos(-11/21)$

b) $w = 2i + 7j + 6\vec{k}$

$u \cdot v = \langle 1, -2, 2 \rangle \cdot \langle 2, 7, 6 \rangle =$

$= 2 - 14 + 12 = 0$

$\Rightarrow$ right angle or $\vec{u} \perp \vec{v}$ or $\vec{u}$ orthogonal to $\vec{v}$

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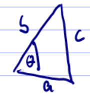
Panel 5

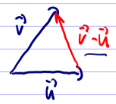
Theorem: If u and v are non-zero vectors in $\mathbb{R}^2$ then

$$\boxed{\frac{u\cdot v}{\|u\|\cdot\|v\|} = \cos\theta} \quad \text{or} \quad \boxed{\vec u\cdot\vec v = \|u\|\|v\|\cos\theta}$$

Law of cosines: $c^2 = a^2+b^2-2ab\cos\theta$



$$\|\vec v-\vec u\|^2 = \|\vec u\|^2+\|\vec v\|^2 - 2\|\vec u\|\|a\|\cos\theta$$



$$(\vec v-\vec u)\cdot(\vec v-\vec u) = \vec u\cdot\vec u + \vec v\cdot\vec v - 2\|\vec u\|\|\vec v\|\cos\theta$$

$$\cancel{vv} - vu - uv + \cancel{uu} = \cancel{uu} + \cancel{vv} - 2\|u\|\|v\|\cos\theta$$

$$v\cdot u = \|u\|\|v\|\cos\theta$$

$$\frac{\vec u\cdot\vec v}{\|\vec u\|\|\vec v\|} = \cos\theta$$

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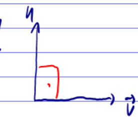
Panel 6

In other words: dot product gives angle between 2 vectors.

$$\vec u\cdot\vec v = \cos(\theta)\,\|\vec u\|\|\vec v\|$$

Thm: $\vec u\cdot\vec v = 0$ if and only if $\vec u$ and $\vec v$ are orthogonal (or perpendicular)



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Panel 7

Ex: Find angle between $u = i-2j+2k =$ and

a) $v = -3i+6j+2k$

done

b) $w = 2i+7j+6\vec k$

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Panel 8

Corollary: Two vectors $\vec v$ and $\vec w$ are perpendicular iff $\vec v\cdot\vec w = 0$

Ex: Which of the following vectors are perpendicular?

a) $\langle 1,2,3\rangle$ and $(-1,-2,-3)$

b) $\langle 1,2,3\rangle$ and $\langle -1,-3,2\rangle$

c) $\langle 1,2,3\rangle$ and $\langle 6,-1,1\rangle$

d) $\langle 1,2,3\rangle$ and $\langle 5,-1,1\rangle$

e) $\langle 1,2,3\rangle$ and $\langle 0,-3,2\rangle$

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Panel 9

Find a vector orthogonal to

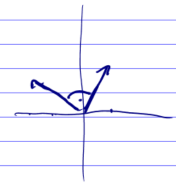
$\langle 5,7\rangle$ $\langle -7,5\rangle$ $\parallel$ $\langle 1,2\rangle$ $\perp$ $\langle -2,1\rangle$ or $\langle 2,-1\rangle$

$\langle 1,3,4\rangle$ $\langle 0,-4,3\rangle$ or

also $\langle -1,-1,1\rangle$ or

$\langle 3,-1,0\rangle$

$\langle 4,0,-1\rangle$



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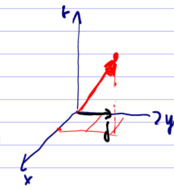
Panel 10

Ex: Find the angle that $\vec{a} = \langle 1,2,3\rangle$ makes with the y-axis:

i.e. angle between $\vec{a}$ and $\vec{j} = \langle 0,1,0\rangle$



$\cos(\theta) = \frac{\vec{a}\cdot\vec{j}}{\|\vec{a}\|\|\vec{j}\|} = \frac{2}{\sqrt{14}}$

$\theta = 58°$

Directional angles:

angle of $\vec{v}$ with x-axis: $\cos(\theta) = v_1/\|v\|$

y-axis: $\cos(\theta) = v_2/\|v\|$

z-axis: $\cos(\theta) = v_3/\|v\|$

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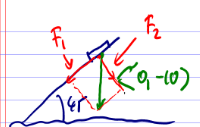
Panel 11



10 kg block sits on an incline of 45°. What force is pulling of block in direction of incline?

$\vec{F} = \vec{F}_1 + \vec{F}_2$. Suppose $\vec{e}_1$ is a unit vector in dir. of $\vec{F}_1$, $\vec{e}_2$ is a unit vector in dir. of $\vec{F}_2$

$\vec{F} = k_1\vec{e}_1 + k_2\vec{e}_2 \quad | \cdot\vec{e}_1$

$\vec{e}_1\cdot\vec{F} = k_1\vec{e}_1\cdot\vec{e}_1 + k_2\vec{e}_2\cdot\vec{e}_1 = k_1$ what I want!

Need $\vec{e}_1 = \frac{1}{\sqrt{2}}\langle -1,-1\rangle$ Answer.

$\vec{e}_1\cdot\vec{F} = \frac{1}{\sqrt{2}}\langle -1,-1\rangle\cdot\langle 0,-10\rangle = \frac{10}{\sqrt{2}}$

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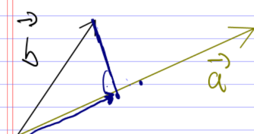
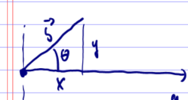
Panel 12

General Question: take two vectors $\vec{a}$ and $\vec{b}$. How much of $\vec{b}$ goes in the direction of $\vec{a}$?



What is the projection (shadow) of $\vec{b}$ onto $\vec{a}$



How long is x: $x = \|\vec{b}\|\cos(\theta) = \|\vec{b}\|\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|}$

Length of proj. of $\vec{b}$ onto $\vec{a}$: $\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|} = \text{comp}_{\vec{a}}(\vec{b})$

Vector of proj. of $\vec{b}$ onto $\vec{a}$: $\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|}\cdot\frac{\vec{a}}{\|\vec{a}\|} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|^2}\vec{a} = \text{proj}_{\vec{a}}(\vec{b})$

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Panel 13

Projection Formula: $\text{proj}_{\vec{a}}\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|^2}\,\vec{a}$

Find length and proj. of $\vec{b} = \langle 1,1,2\rangle$ onto $\vec{a} = \langle -2,3,1\rangle$

$$\text{proj}_{\vec{a}}(\vec{b}) = \frac{a\cdot b}{\|a\|^2}\,\vec{a} = \frac{\langle -2,3,1\rangle\cdot\langle 1,1,2\rangle}{14}\cdot\langle -2,3,1\rangle$$

$$= \frac{-2+3+2}{14}\cdot\langle -2,3,1\rangle = \frac{3}{14}\langle -2,3,1\rangle$$

Length of $\text{proj}_{\vec{a}}(\vec{b}) = \left\|\frac{3}{14}\langle -2,3,1\rangle\right\| = \frac{3}{14}\cdot\left\|\langle -2,3,1\rangle\right\| = \frac{3}{14}\cdot\sqrt{14} = \frac{3}{\sqrt{14}}$

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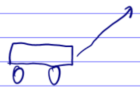
Panel 14

Application: A wagon is pulled a distance of 100 m by a constant force of 70 N, applied to a handle held at 35°. Find work done by F.



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Panel 15

1/2

Calc 3 - Quiz #1

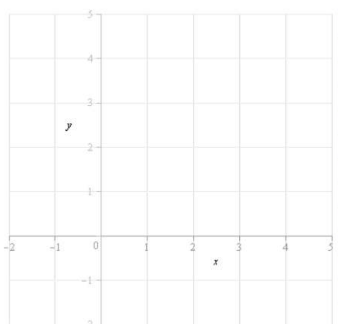
① Consider the vector $\vec{v} = \langle 1,-1\rangle$ and $\vec{w} = \langle 2,3\rangle$

a) Draw the vectors

b) Draw $\vec{v}+\vec{w}$

c) Find algebraically:

$3\vec{v} - 2\vec{w}$



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Panel 16

2/2

② If $\vec{w} = \langle -2,6,3\rangle$, find

a) $\|\vec{w}\|$

b) a unit vector in the direction of $\vec{w}$

③ If $\vec{v} = \langle 1,-2,3\rangle$ and $\vec{w} = \langle 3,1,0\rangle$, find

$\vec{v}\cdot\vec{w}$

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