Panel 1

Last time

Review of Calc 1 + 2 — limit, contin., diff., integral

Intro to $\mathbb{R}^3$

Points, coord. systems

Distance: $d=\sqrt{x^2+y^2+z^2}$ dist$(P(x,y,z),(0,0,0))$

Objects: sphere

sheets

1

Panel 2

Quiz next Wed

Def: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$ is sphere ~~cloche~~ centered at $P(x_0,y_0,z_0)$ with radius r.

Ex: Find the center + radius of the sphere

$x^2+y^2+z^2+10x+4y+2z-19=0$

$x^2+10x+25-25+y^2+4y+4-4+z^2+2z+1-1=19$

$(x+5)^2+(y+2)^2+(z+1)^2=19+25+4+1$

$=49$

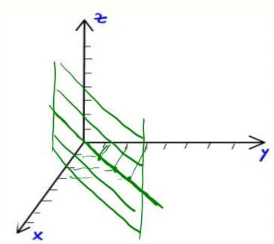
sphere centered at $(-5,-2,-1)$ with radius 7

2

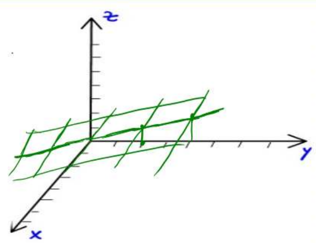
Panel 3

3D Objects

a) $y=x$



b) $z=\frac{1}{2}y$



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Panel 4

Drawing 3D objects with Maple

Maple can easily draw 3D objects

Start Maple.

```
> with(plots);
> implicitplot3d(z = y^2, x = -3..3, y = -3..3, z = -1..9);
> plot3d(x^2, x = -3..3, y = -3..3);
> implicitplot3d(z^2 + y^2 = 4, x = -3..3, y = -3..3, z = -3..3);
```

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Panel 5

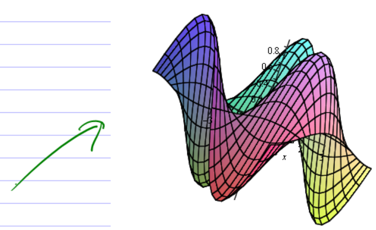
Ex: Use Maple to graph the following:

(use "Insert | Screen Grab" to paste into panel)

$x^2+y^2+z^2=4$

$y^2+z^2=2$

$z=\sin(x)\cdot\cos(y)$



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Panel 6

Vectors

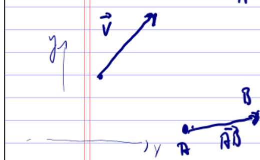
Understand points in 3D (and 2D). Want to investigate more general objects => vectors.

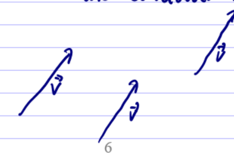
Def: A vector is a directed line segment, i.e. a part of a line that has a length and a direction.

They are written as $\vec{v}, \vec{u}, \vec{w}$, etc..

A vector from A to B is $\vec{v}=\vec{AB}$

All vectors with same length and direction are considered the same





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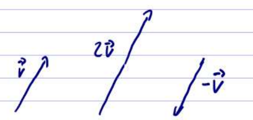
Panel 7

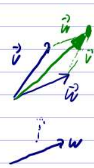
Vector Math, Geometrically

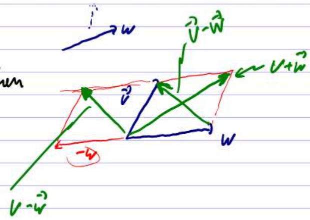
If v is a vector then $c\in\mathbb{R} \Rightarrow c\vec{v}$ is c-times as long as $\vec{v}$. If $c<0$, it reverses $\vec{v}$

If v, w are vectors, then $v+w$ is a vector that is the diagonal of the parallelogram $\vec{v}, \vec{w}$

If v, w are vectors, then $\vec{v}-\vec{w}$ is $\vec{v}+(-\vec{w})$







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Panel 8

Vector Math, Algebraically

Algebraically v is described by components:

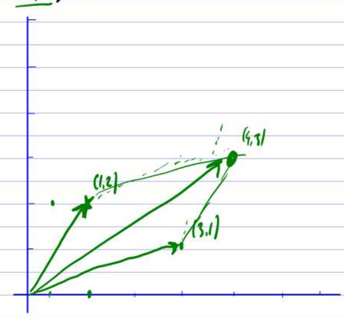
$\vec{v}=\langle v_1, v_2\rangle$ or $\vec{v}=\langle v_1, v_2, v_3\rangle$

Ex: Suppose $\vec{v}=\langle 1,2\rangle$, $\vec{w}=\langle 3,1\rangle$. Find

$\vec{v}+\vec{w}$ $=\langle 1,2\rangle+\langle 3,1\rangle=$ $=\langle 4,3\rangle$

$\vec{v}+2\vec{w}$ $=\langle 1,2\rangle+2\langle 3,1\rangle=$ $=\langle 1,2\rangle+\langle 6,2\rangle=$ $=\langle 7,4\rangle$

$3\vec{v}-\vec{w}$ $=\langle 3,6\rangle-\langle 3,1\rangle=$ $=\langle 0,5\rangle$



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Panel 9

Vectors: Some Definitions

Def: The Length or norm of a vector $\vec{v} = (v_1, v_2)$ is: $\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$ or $\sqrt{v_1^2 + v_2^2 + v_3^2}$

Def: A unit vector $\vec{v}$ is a vector such that $\|\vec{v}\| = 1$

Note: If $\vec{v} = (v_1, v_2)$ is any non-zero vector, then $\vec{u} = \frac{1}{\|\vec{v}\|} \vec{v}$ $\left(\| \frac{1}{\|\vec{v}\|} \vec{v} \| = \| \frac{1}{c}(v_1, v_2) \| = \sqrt{\frac{1}{c^2}v_1^2 + \frac{1}{c^2}v_2^2} = \frac{1}{c}\sqrt{v_1^2 + v_2^2} = c \cdot \frac{1}{c} = 1 \right)$ is a unit vector pointing in the same direction as $\vec{v}$.

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Panel 10

Ex: $\vec{v} = \langle \frac{1}{2}, \frac{3}{4} \rangle$, $\vec{w} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$. Which one is a unit vector?

$\|v\| = \sqrt{1/4 + 9/16} \neq 1$ no

$\|w\| = \sqrt{1/2 + 1/2} = 1$ yes

Ex: Find unit vector in the direction of $\vec{v} = (1, -5)$ and $\vec{w} = \langle 3, 2, -1 \rangle$

$\vec{v} = \langle 1, -5 \rangle \Rightarrow \|v\| = \sqrt{26} \Rightarrow \vec{u}_{\vec{v}} = \frac{1}{\sqrt{26}} \langle 1, -5 \rangle$

$\vec{u}_{\vec{w}} = \frac{1}{\sqrt{14}} \langle 3, 2, -1 \rangle$

Some vectors are special.

$i = \langle 1, 0, 0 \rangle$
$j = \langle 0, 1, 0 \rangle$
$k = \langle 0, 0, 1 \rangle$

basic unit vectors

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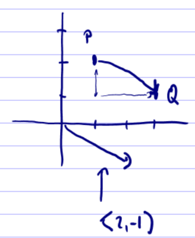
Panel 11

Ex: If $\vec{v} = -2\vec{i} + 3\vec{k}$ write $\vec{v}$ in standard notation and find $\|\vec{v}\|$

$\boxed{\vec{v} = \langle -2, 0, 3 \rangle} = -2\underset{i}{\langle 1, 0, 0 \rangle} + 0 \cdot \underset{j}{\langle 0, 1, 0 \rangle} + 3\underset{k}{\langle 0, 0, 1 \rangle}$, $\|v\| = \sqrt{13}$

Note: If $P(1, 2)$ and $Q(3, 1)$ find $\vec{v} = \vec{PQ}$:



$\vec{PQ} = \langle 3-1, 1-2 \rangle = \langle 2, -1 \rangle$

"Q - P"

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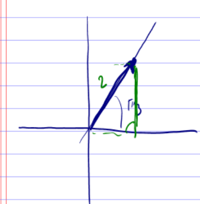
Panel 12

Other ways to describe vectors: Find a vector of length 2 that makes an angle of $\pi/3$ with positive x-axis.



$x = 2\cos(\pi/3) = 1$

$y = 2\sin(\pi/3) = \sqrt{3}$

$\vec{v} = \langle x, y \rangle = \langle 1, \sqrt{3} \rangle$

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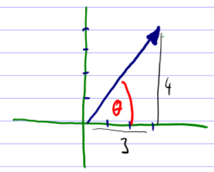
Panel 13

Other way around: find angle that $\vec{v} = 3i + 4j$ makes with the positive x-axis.

$\vec{v} = \langle 3, 4 \rangle$

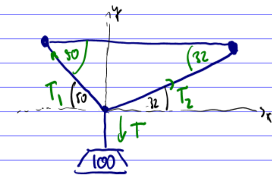


$\tan(\theta) = \frac{4}{3} \Rightarrow \theta = \arctan(4/3) = \underline{53°}$

$\|v\| = 5$

Panel 14

What are vectors good for: a 100 lb weight hangs from two wires as shown. Find the forces T_1 and T_2 acting on the wires and their magnitudes.



$T_1 = \langle -\|T_1\| \cos(50), \|T_1\| \sin(50) \rangle$

$T_2 = \langle \|T_2\| \cos(32), \|T_2\| \sin(32) \rangle$

Know: $T_1 + T_2 = \langle 0, 100 \rangle$

$-\|T_1\| \cos(50) + \|T_2\| \cos(32) = 0 \Rightarrow \|T_2\| = \|T_1\| \frac{\cos(50)}{\cos(32)}$

$\|T_1\| \sin(50) + \|T_2\| \sin(32) = 100$ ✓

Panel 15

$-\|T_1\| \cos(50) + \|T_2\| \cos(32) = 0 \Rightarrow \|T_2\| = \|T_1\| \frac{\cos(50)}{\cos(32)}$

$\|T_1\| \sin(50) + \|T_2\| \sin(32) = 100$ ✓

$\|T_1\| \sin(50) + \|T_1\| \frac{\cos(50)}{\cos(32)} \sin(32) = 100$

$\|T_1\| \left(\sin(50) + \frac{\cos(50)}{\cos(32)} \sin(32) \right) = 100$

$\|T_1\| = 85.64$ lb, $\|T_2\| \approx 64.91$

$\underline{T_1 = \langle -55.05, 65.6 \rangle}$

$\underline{T_2 = \langle 55.05, 34.4 \rangle}$

Panel 16

Know how to add / subtract vectors.

$v \cdot w = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = \langle v_1 w_1, v_2 w_2 \rangle$ $\boxed{\cancel{\text{No good}}}$

$\langle 1, 0 \rangle \cdot \langle 0, 5 \rangle = \langle 0, 0 \rangle$, $v \cdot w = 0$ but neither v, w are zero

Dot Product: 2D $v \cdot w = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = v_1 w_1 + v_2 w_2$

vectors scalar or number

3D $v \cdot w = \langle v_1, v_2, v_3 \rangle \cdot \langle w_1, w_2, w_3 \rangle = v_1 w_1 + v_2 w_2 + v_3 w_3$

dot

Panel 17

Examples of Dot Product

① $\langle 3,5\rangle\cdot\langle -1,2\rangle = -3 + 10 = 7$

② $\langle 2,3\rangle\cdot\langle -3,2\rangle = -6 + 6 = 0$

③ $\langle 1,-3,4\rangle\cdot\langle 1,5,2\rangle = 1 - 15 + 8 = -6$

next: what does it mean geometrically?

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