Panel 1

Math 2411/2511 : Calculus 3

Kindle -

More technology: Dy/know

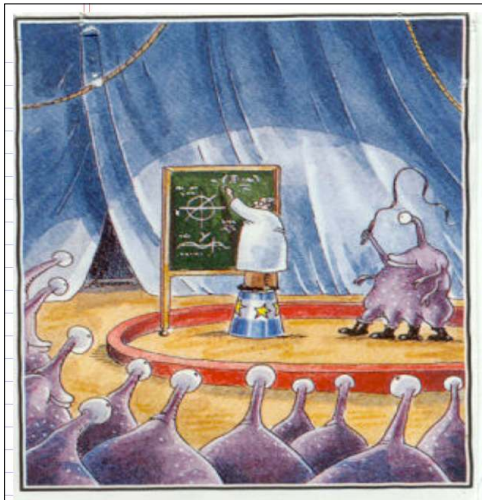
-> homepage

username: 8 letters

password: username again -> Try it as HW

1

Panel 2



Abducted by an alien circus company, Professor Doyle is forced to write calculus equations in center ring.

2

Time to get SERIOUS - Time for some Mathematics

Panel 3

Calc 1 - Overview

$\lim_{x\to 2} x^2 = 4$ Geometry

$\lim_{x\to 0} \frac{\sin(x)}{x}$ How - to

	Def.	Geometry	How - to
• Limit $\lim_{x\to x_0} f(x) = L$	For every $\varepsilon > 0$ there is $\delta > 0$ s.t. if $\lvert x - x_0\rvert < \delta$ then $\lvert f(x) - L\rvert < \varepsilon$	as x gets close to x_0, $f(x)$ gets close to L	plug in, hope for the best, l'Hospital
• Continuity	$\lim_{x\to x_0} f(x) = f(x_0)$	no holes or gaps	polyn are cont. rationals are cont. where defined
• Differentiation $f'(x)$	$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$	slope of tangent	product quotient chain } rule
• Integration	$\lim_{n\to\infty} \sum_{i=0}^{n} f(x_i)\,\Delta x_i$	area under curve if $f \geq 0$	Fund. thm. of calculus

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Panel 4

Calc 1 : Some Refresher

① a) $\lim_{x\to 1} \frac{x^2-1}{x} = 0$ b) $\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$ c) $\lim_{x\to 0} \frac{x^2 \sin(x)}{1-\cos(x)} =$

② $f(x) = x^2 - \sin(x) + e^x - \sqrt{x} + \ln(x)$ ($\sqrt{x} = x^{1/2}$)

$f'(x) = 2x - \cos(x) + e^x - \frac{1}{2}x^{-1/2} + \frac{1}{x}$

$g(x) = \frac{x \sin(x)}{\sqrt{1-x^2}}$, $g'(x) = \frac{(\sin(x) + x\cos(x))\sqrt{1-x^2} - x\sin(x)\left(\frac{1}{2}(1-x^2)^{-1/2}(-2x)\right)}{(\sqrt{1-x^2})^2}$ ✓

③ $\int 5x - 3\sqrt{x} + e^x - \frac{5}{x} + \frac{6}{x^2}\,dx =$ ($\sqrt{x} = x^{1/2}$, $\frac{1}{x^2} = x^{-2}$)

$= 5\cdot\frac{1}{2}x^2 - 3\cdot\frac{2}{3}x^{3/2} + e^x - 5\ln(x) - 6(-1)x^{-1} + C$

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Panel 5

$$\lim_{x\to 0}\frac{x^2\sin(x)}{1-\cos(x)} = \lim_{x\to 0}\frac{2x\sin(x)+x^2\cos(x)}{\sin(x)}$$

$$= \lim_{x\to 0}\frac{2\sin(x)+2x\cos(x)+2x\cos(x)-x^2\sin(x)}{\cos(x)} = \frac{0}{1} = 0$$

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Panel 6

Calc 2 - Overview

- Heavy Duty Integration Techniques
  - eg: int. parts.
- Applications of Integration
  - eg.. arc length,
- Sequences & Series
  - eg: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$
- Differential Equations
  - eg. sep. $f(x)\,g(y) = y'$

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Panel 7

Calc 2 Refresher

① $\int x\cdot\sin(x)\,dx = -x\cos(x) + \int\cos(x)\,dx = -x\cos(x)+\sin(x)+C$

$f' = \sin(x)$, $f = -\cos(x)$; $g = x$, $g' = 1$

② $\int \frac{5x}{x^2-x-2}\,dx = \int \frac{10/3}{x-2} + \frac{5/3}{x+1}\,dx$ $\quad \frac{5x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$

$= \frac{10}{3}\ln|x-2| + \frac{5}{3}\ln|x+1| + C$ $\quad A(x+1)+B(x-2) = 5x$

let $x=-1$: $-3B = -5$, $B = 5/3$

$x=2$: $3A = 10$, $A = 10/3$

③ $\frac{5x}{1+x^2}$ as a power series

HW geometric series

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Panel 8

Calc 3

Calc 3 ≅ calc 1 + calc 2 + several variables

Explore $\mathbb{R}^3$

Vector-valued functions
lim., cont., diff

Functions of two variables
lim., cont., diff,

Multiple Integrals

Icing on the cake

Applications

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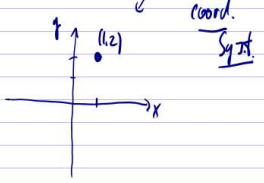
Panel 9

Introducing $\mathbb{R}^3$

Cartesian coord. Syst.

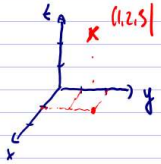
Coordinate system in $\mathbb{R}^2$:

Ex: plot $(1,2)$, $(4,3)$



Coordinate system in $\mathbb{R}^3$:

Ex: plot $(1,1,1)$, $(1,2,3)$



x-axis: all points $(x, 0, 0)$

y-axis:

z-axis:

xy plane: all points $(\ ,\ , 0)$

yz plane: $(0,\ ,\ )$

xz plane: $(\ , 0, \ )$

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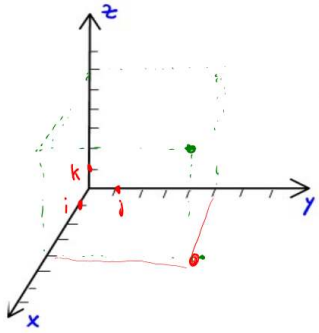
Panel 10

Ex: Plot the following Points.



$P(3,2,1)$ - use blue ✓

$Q(4,5,6)$ - use green ✓

$I = (1,0,0)$

$J = (0,1,0)$

$K = (0,0,1)$

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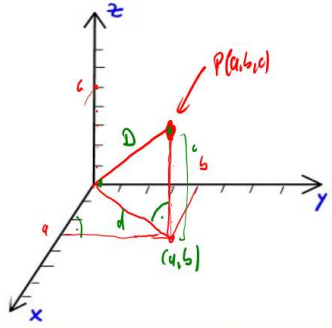
Panel 11

Distance in $\mathbb{R}^3$



$P(a,b,c)$

Distance between origin and P

$d = \sqrt{a^2+b^2}$

$D = \sqrt{d^2+c^2} = \sqrt{a^2+b^2+c^2}$

Distance between $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ is

$D = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

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Panel 12

$P(3,2,1)$ and $Q(4,5,6)$

Find distance to origin and distance P to Q

$\text{dist}(P,O) = \sqrt{9+4+1} = \sqrt{14}$, $\text{dist}(Q,O) = \sqrt{16+25+36}$

$\text{dist}(P,Q) = \sqrt{(3-4)^2+(2-5)^2+(1-6)^2} =$

$= \sqrt{1+9+25}$

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Panel 13

3D Objects

$P(x,y,z) \Rightarrow d = \sqrt{x^2+y^2+z^2} \Rightarrow d^2 = x^2+y^2+z^2$

Collection of all points that are d-units away from origin

Sphere: $d^2 = x^2+y^2+z^2$, center $(0,0,0)$, radius d

$d^2 = (x-x_0)^2+(y-y_0)^2+(z-z_0)^2$, center (x_0,y_0,z_0), radius d

Ex: $x^2+y^2+z^2-2x-4y+8z+17=0$

$x^2-2x+1(-1)+y^2-4y+4(-4)+z^2+8z+16(-16) = -17$

$(x-1)^2+(y-2)^2+(z+4)^2 = -17+16+4+1$

$= 4$

center $(1,2,-4)$, radius 2

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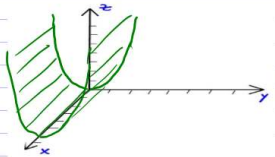
Panel 14

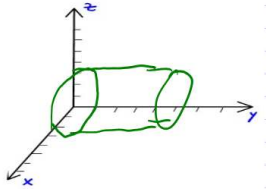
Objects with 2 Variables in 3D

Graph of object with 2 variables in 3D space is a "sheet":

Ex: $z = y^2$



Ex: $x^2+z^2 = 1$



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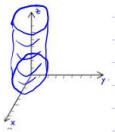
Panel 15

Quick Quiz

1

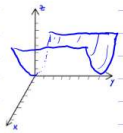


✓ a) $x^2+y^2=1$

b) $x^2+z^2=1$

c) $y^2+z^2=1$

2

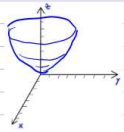


a) $y=x^2$

✓ b) $z=x^2$

c) $z=y^2$

3



a) $z=xy$

b) $z=x^2-y^2$

✓ c) $z=x^2+y^2$

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