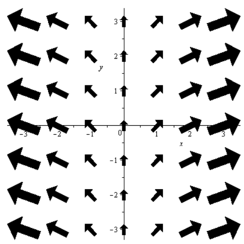
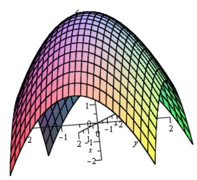
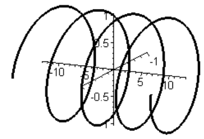
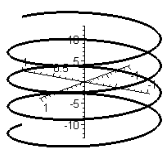
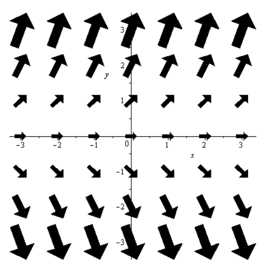
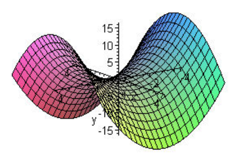
Calc 3 – Final Exam

This is a take-home, open-book, open notes exam; you even may use Maple to assist you in calculations. You must, however, complete it *entirely on your own*. It is due on the last day of exams – no exception. Please indicate clearly where each problem starts and do not forget to put your name on your exam.

1. Please state the following:
   a) an equation relating the dot product of two vectors with the angle between them
   b) the definition of the gradient of a function $f(x, y, z)$ and its properties
   c) the main difference between Green's and Stoke's Theorem as far as the vector field $\vec{F}$ is concerned
   d) the Divergence Theorem (also known as Gauss' Theorem)

2. Match the following pictures with the algebraic expressions below.

[A]  [B]  [C] 

[D]  [E]  [F] 

(1) $f(x, y) = 6 - x^2 - y^2$ (2) $f(x, y) = y^2 - x^2$ (3) $r(t) = \langle \cos(t), \sin(t), t \rangle$

(4) $r(t) = \langle \cos(t), t, \sin(t) \rangle$ (5) $F(x, y) = \langle x, 1 \rangle$ (6) $F(x, y) = \langle 1, y \rangle$

3. Determine if the plane through the points $P(1,0,0)$, $Q(0,2,0)$, and $R(0,0,3)$ is perpendicular to the plane given by the equation $2x - 2y - 3z = 1$

4. A baseball is hit 4 feet above ground at an initial velocity of $< 80, 80 >$ feet per second. Find the maximum height reached by the baseball. Will it clear a 15-foot high fence located 350 feet from home base?

5. Determine the following limits, if possible, or explain why they don't exist.

$\lim_{(x,y)\to(0,0)} \frac{x+2y+3}{x^2+2y^2+3}$ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

6. Find all critical points and test them for relative extrema for the function $f(x,y) = -x^3 + 3xy - \frac{3}{2}y^2$ (*Hint: There are two critical points*)

7. Evaluate the following integrals:

   a) $\iint_R e^{x^2}\, dA$, where R is the triangular region bounded by $y = 0$, $y = x$, and $x = 1$

   b) $\int_C x^2 + y\text{^}2\, ds$, where C is the curve given by $r(t) =< 2\cos(t), 2\sin(t) >$ for $0 \le t \le \pi$

   c) $\int_C \vec{F}\, d\vec{r}$, where $\vec{F}(x,y) =< -y, x >$ and C is the line segment from $P(-1,-1)$ to $Q(2,3)$

   d) The flux of the vector field $\vec{F}(x,y,z) =< x, y, z >$, where S is the portion of the surface $z = 10 - 2x - 2y$ between the coordinate planes.

8. For the following integrals there are at least two ways to evaluate them. Use the most convenient method and quote the appropriate theorem.

   a) $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y,z) =< 2xy^3 + x\cos(x), 3x^2y^2 - \sin(y)e^y >$ and C is the closed curve given by the boundary of the square with corner points (-1,-1), (-1,1), (1, -1), and (1,1).

   b) $\int_C (3x^2y - y^2)dx + x^3 dy$ where C is the closed curve given by the boundary of the triangle with corner points (0,0), (0,1), and (1, 0), oriented counter-clockwise.

   c) $\iint_S \vec{F} \cdot \vec{n}\, dS$ where $F(x,y,z) =< x, 2y, 3z >$ and S is given by $x^2 + y^2 + z^2 = 9$

   d) $\int_C \vec{F}\, dr$ where $F(x,y,z) =< z^2, x^2, y^2 >$ and C is the boundary of the surface S given by $z = 1 - x - y$, oriented counter-clockwise.