Note Title 10/30/2008

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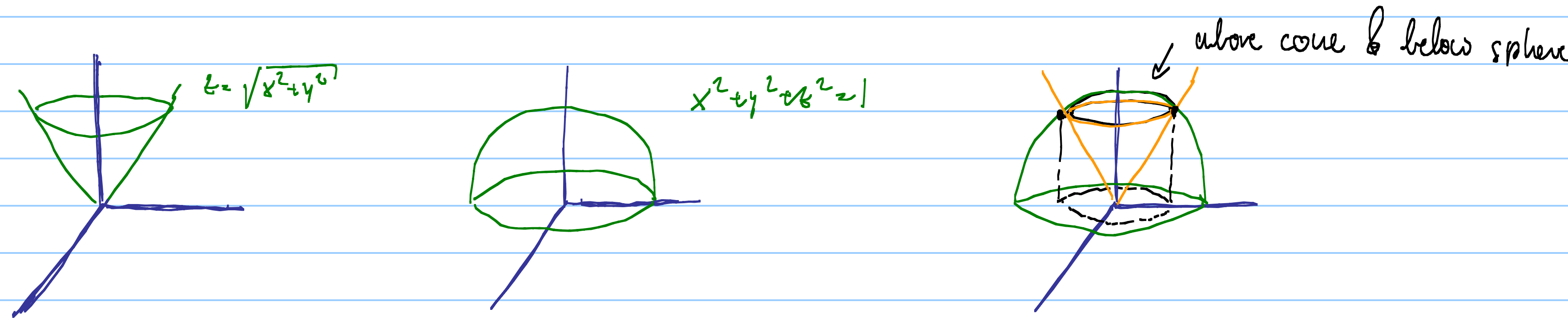
Find the volume above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = 1$



First find intersection: $x^2+y^2+(x^2+y^2) = 1 \iff 2x^2+2y^2=1 \iff x^2+y^2 = \frac{1}{2}$

$\Rightarrow$ volume is above disk of radius $\frac{1}{\sqrt{2}}$ bounded below by cone and above by sphere

$\Rightarrow$ add volumes of height (sphere - cone) above disk of radius $\frac{1}{\sqrt{2}}$:

$$\Rightarrow \quad V = \iint_D \sqrt{1-x^2-y^2} - \sqrt{x^2+y^2} \, dA \quad , \text{ where } D = \{ x^2+y^2 \varepsilon 1/\sqrt{2} \}$$

convert to polar: $$V = \int_0^{2\pi} \int_0^{1/\sqrt{2}} \left(\sqrt{1-r^2} - r \right) r \, dr \, d\theta = \frac{2}{3}\pi \left(1 - \frac{1}{\sqrt{2}} \right) \quad \text{(Maple)}$$

For exercise, let's work out the integral by hand:

$$V = \int_0^{2\pi} \int_0^{1/\sqrt{2}} \left(\sqrt{1-r^2} - r \right) r \, dr \, d\theta = 2\pi \int_0^{1/\sqrt{2}} r\sqrt{1-r^2} - r^2 \, dr = 2\pi \left[\left(-\frac{1}{2}\right)\frac{2}{3} \left(1-r^2\right)^{3/2} \Big|_0^{1/\sqrt{2}} - \frac{1}{3} r^3 \Big|_0^{1/\sqrt{2}} \right] =$$

$$= 2\pi \left[-\frac{1}{3} \left[\left(\frac{1}{2}\right)^{3/2} - 1 \right] - \frac{1}{3} \left(\frac{1}{2}\right)^{3/2} \right] = \frac{2}{3}\pi \left(1 - \left(\frac{1}{2}\right)^{3/2} - \left(\frac{1}{2}\right)^{3/2} \right) =$$

$$= \frac{2}{3}\pi \left(1 - \frac{2}{2^{3/2}} \right) = \frac{2}{3}\pi \left(1 - \frac{\cancel{2}}{\cancel{2} \cdot 2^{1/2}} \right) = \frac{2}{3}\pi \left(1 - \frac{1}{\sqrt{2}} \right)$$