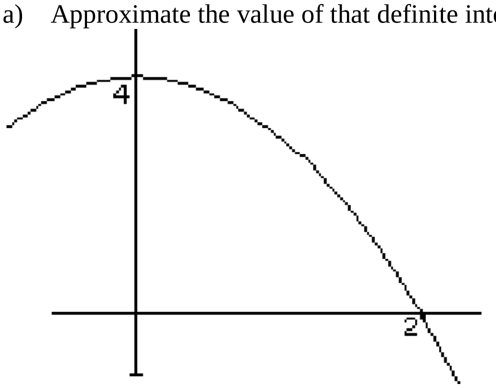
Practice Exam 3

Consider a definite integral of the form $\int_a^b f(x)dx$

a) What is the exact mathematical definition of that definite integral?
b) What is the geometric interpretation of that definite integral?
c) What is the definition of the average value of a function over the interval [a, b]
d) What is the geometric interpretation of the average value of a function over the interval [a, b]
e) What is the MVT for Integrals
f) State the first fundamental theorem of calculus, as it applies to that integral. What is that theorem good for, in your own words?
g) What is the definition of the natural log function *ln(x)*
h) What is the difference between $\int_a^b f(x)dx$ and $\int f(x)dx$

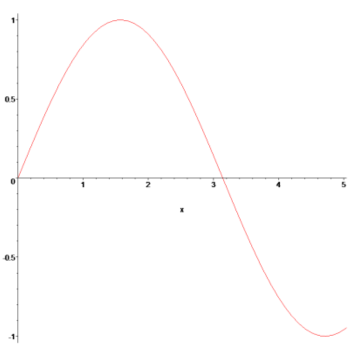
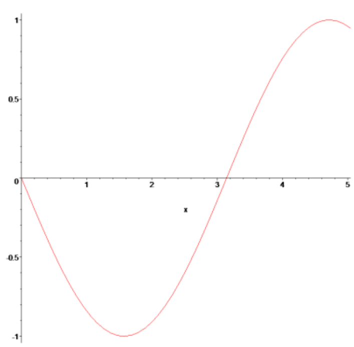
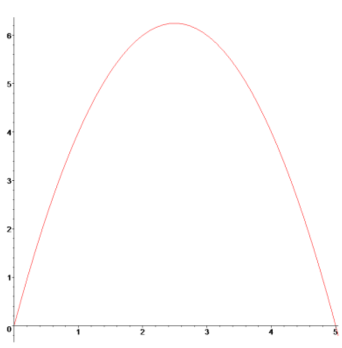
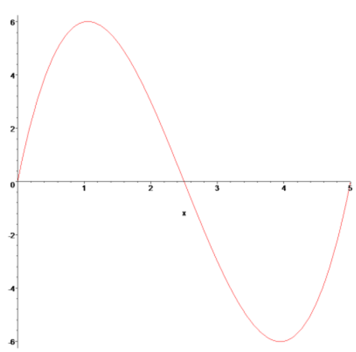
Consider the following definite integral: $\int_0^2 4 - x^2 dx$ (The integrand is depicted below).

a) Approximate the value of that definite integral by using 4 subdivisions and <u>right</u> rectangles in the corresponding summation.



b) Find the exact value of that definite integral by using the first fundamental theorem of calculus.

Below are the graphs of three functions. For each graph, decide whether $\int_0^5 f(x)dx$ is positive, negative, or zero.

Evaluate each of the following integrals. If you use substitution, please indicate clearly what your ***u*** is, and what the corresponding ***du*** is.

$\int 3x^2 - \frac{5}{x^3} + \sqrt[3]{x} - 2\cos(x)dx$ $\int x^2 + \frac{1}{\sqrt{x}} + \pi^2 dx$ $\int \frac{3x^2}{\sqrt{x^3+1}}dx$

$\int \frac{2}{x}dx$ $\int \tan(x)dx$ $\int x\sqrt{x-1}dx$

$\int 5x\sqrt{x^2+1}dx$ $\int \frac{\sin(x)}{\cos^3(x)}dx$

$\int \frac{\cos(2x)}{\sin^3(2x)}dx$ (Hint: use ***u = sin(2x)***) $\int \frac{1}{x\ln(x)}dx$ $\int (x+\frac{1}{x})^2 dx$

Evaluate each of the following definite integrals.

$\int_0^{\pi/2} \cos(x)dx$ $\int_1^2 x(x-1)dx$ $\int_{-3}^3 \sqrt{9-x^2}dx$ (trick question)

$\int_1^4 \frac{x-1}{\sqrt{x}}dx$ $\int_1^2 \frac{2-t^3}{t^2}dt$ $\int_{-1}^1 \frac{x}{\sqrt{x^2+2}}dx$

$\int_\pi^\pi \cos(4x^2)dx$ (trick question) $\int_{-1}^1 \frac{1}{x+2}dx$ $\int_0^1 \frac{x-1}{x+1}$

At SLAC (Stanford Linear Accelerator) the initial position of a particle was recorded at time ***t = 0*** to be 10m. Several detectors were used to record the speed of the particle, and it could be determined that the velocity function of the particle was given by ***v(t) = 4t^2^ + 3t***. What is the distance function of the particle, and where is the particle after 5 seconds?

The probability that a person will remember between a% and b% of material learned in a certain experiment is $P_{a,b} = \int_a^b \frac{15}{4}x\sqrt{1-x}dx$.

For a randomly chosen individual, what is the probability that he or she will recall between 50% and 70% of the material?

Suppose that gasoline is increasing in price according to the equation $p = 1.2 + 0.04t$ where *p* is the dollar price per gallon, and *t* is the time in years, with *t = 0* representing 1990. If an automobile is driven 15,000 miles a year, and gets M miles per gallon, the annual fuel cost is $C = \frac{15000}{M}\int_t^{t+1} p\,dt$. Estimate the annual fuel cost (a) for the year 2000 and (b) for the year 2005.

Define a function $S(x) = \int_0^x \cos^2(t^2)dt$. Then find S(0), S'(x), and S"(x). Where is the function increasing and decreasing? If $G(x) = \int_x^{x^2} \sin(t^2)dt$ then find G(1) and G'(x).

Find the derivatives of the following functions:

$f(x) = \ln\left(\frac{\sqrt{x-1}}{(x-1)^2}\right)$ $f(x) = (x-1)^2\ln\left(\sqrt{x^2-1}\right)$ $f(x) = \frac{(x-1)^2}{(x+1)^3}(x+2)^4$

Find the following areas:

- Area under $f(x) = x^2 + 1$ from $x = -1$ to $x = 2$
- Area between $f(x) = 4 - x^2$ and x-axis
- Area under $f(x) = x^2 - 1$ from $x = 0$ to $x = 2$

Prove that $\int_0^1 2xdx = 1$, using the definition of the integral

Prove that *ln(x)* is increasing for $x > 0$

Prove that *ln(x b) = ln(x) + ln(b)*

Prove the MVT for integrals, using the Fund. Theorem of Calc and the ordinary MVT