

Find the Taylor series for the following functions, complete with radius of convergence, centered at the origin.

1. e^{x^2}
2. $x \sin(x)$
3. $x^2 \cos(x^3)$
4. $e^x \sin(x)$ (first three terms only)
5. $\tan(x)$ (first three terms only)
6. $\int \cos(t^2) dt$
7. $\int e^{-x^2} dx$
8. $\int x e^{x^2} dx$

Solve the following differential equations:

1. $y' = 5 \sin(x)$, $y(0) = 5$
2. $y' = \frac{2x}{y}$
3. $y' = 2y$, $y(0) = 4$
4. $\frac{dy}{dx} = 4 - x$
5. $\frac{dy}{dx} = 4 - y$
6. $y' = \frac{\sqrt{x}}{3y}$
7. $y' = x(1 + y)$

The rate of change of y is proportional to y . When $t = 0$, $y = 2$. When $t = 2$, $y = 4$. What is the value of y when $t = 3$?

The number of bacteria in a certain culture increases from 600 to 1800 in 2 hours. Assuming that the exponential law of growth holds, find a formula for the number of bacteria in the culture at any time t . What is the number of bacteria at the end of 4 hours?

Radium decays exponentially and has a half-life of approximately 1600 years. That is, given any quantity, one-half of it will disintegrate in 1600 years. Find a formula for the amount $q(t)$ remaining from 50 mg of pure radium after t years. When will there be 20 mg left?