Math 1511 – Practice for Final Exam

Exam layout: The final exam has questions according to the following categories:

- Question 1: Definitions
- Question 2: True/False questions and/or Picture Problems
- Question 3: Area between curves
- Question 4: Volume of a rotational solid
- Question 5: Arc Length of a function y = f(x)
- Question 6: Evaluate integrals (multi-part, **at most** 5 parts, like: simple subst, int by parts, partial fraction decomposition, trig substitution, indefinite integral
- Question 7: limit (possibly multi-part, one easy limit, one with l'Hospital, and one tricky with l'Hopital)
- Question 8: Series (multipart, 2 questions about convergence, one about power series)
- Question 9: Differential equation (multipart – 2 different types of DE)
- Question 10: Exp. Growth and decay 'story' problem
- Question 11: Parametric equations, slope of tangent, length of parametric curve
- Extra Credit Question: SOMETHING …

Sample Questions:

There are many sample questions below, many more than will be on the final. Make sure you can do *at least one or two* of every type of question.

1. Please state the definitions of the following terms

a) The area between two functions
b) Volume of a rotational solid, by disks or shells
c) Integration by Parts, Partial Fraction Decomposition, Trigonometric Substitution
d) Improper Integrals
e) Length of a curve f(x) between a and b
f) L'Hospital's Rule
g) What is an "infinite sequence", an "infinite series", or the N-th partial sum
h) The series $\sum_{n=0}^{\infty} a_n$ converges to the limit *L*
i) What is the Divergence Test? Ratio test? Limit comparison. Test?
j) What is a Power Series?
k) Differential equation, differential equation with initial condition
l) Separable differential equation, first-order differential equation

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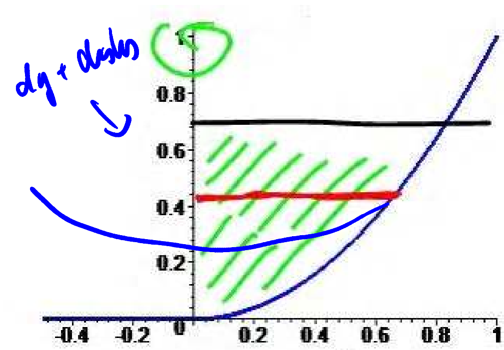
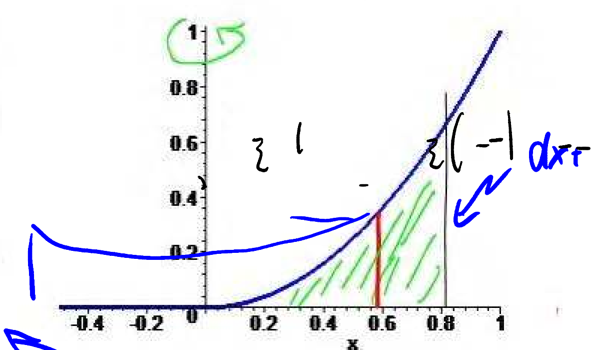
Below are two pictures, indicating sketches of solids of revolution around *x*-axis. Each contains a red "slice" used to compute the solid's volume. Match picture to integral by connecting them with a line.





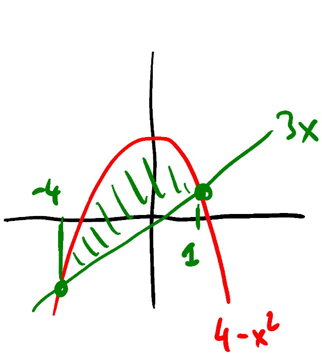
$$\int_a^b 2\pi x f(x)\,dx \qquad\qquad \int_a^b \pi[f(y)]^2\,dy$$

s   disks

Decide which method to use by drawing lines from an integral to the corresponding method. You do NOT have to actually find the integral.

$\int x^3 e^x dx$	Simple Substitution Rule
$\int x^2 e^{x^3} dx$	Integration by Parts once
$\int x \sin(x) dx$	Integration by Parts twice or more
$\int \sin(x) e^x dx$	Integration by Parts followed by solving for an integral

Find the area bounded by the curves $f(x) = 4 - x^2$ and $g(x) = 3x$.



$4 - x^2 = 3x$

$\Leftrightarrow 0 = x^2 + 3x - 4$

$= (x+4)(x-1)$

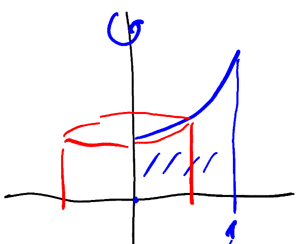
$x = -4, 1$

$A = \int_{-4}^{1} 4 - x^2 - 3x \, dx =$

$= 4x - \frac{1}{3}x^3 - \frac{3}{2}x^2 \Big|_{-4}^{1} = 2$

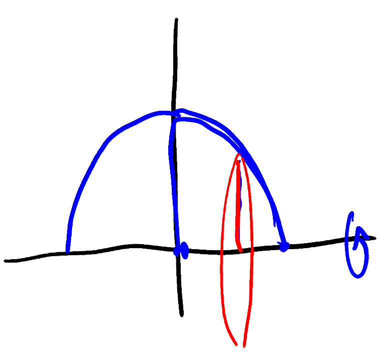
$= 4 - \frac{1}{3} - \frac{3}{2} + 16 - \frac{1}{3} \cdot 64 + \frac{3}{2} 16 = \frac{125}{6}$

Find the volume of the solid generated by revolving the plane region bounded by $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ around the y-axis. Use any method you like.



$V = \int_0^1 2\pi x (x^2+1) dx = 2\pi \left(\frac{1}{4}x^4 + \frac{1}{2}x^2 \Big|_0^1 \right) = 2\pi \left(\frac{1}{4} + \frac{1}{2} \right) = \frac{6\pi}{4} = \underline{\underline{\frac{3\pi}{2}}}$

Find the volume of the solid generated by revolving the plane region bounded by $f(x) = 1 - x^2$ around the x-axis, where $0 \le x \le 1$. Use any method you like.



$V = \int_0^1 \pi [1 - x^2]^2 dx = \pi \int_0^1 1 - 2x^2 + x^4 dx =$

$= \pi \left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \Big|_0^1 \right) =$

$= \underline{\pi \left(1 - \frac{2}{3} + \frac{1}{5} \right)}$

Integrate using any method:

$\int (3x-2)^5 dx$ $\quad \frac{1}{6}\cdot\frac{1}{3}(3x-2)^6 + C$

$\int \sin(x) e^{\cos(x)} dx$ $\quad = -e^{\cos(x)} + C$ $\quad$ (subst $u = \cos(x)$)

$\int \frac{4x}{x^2+9} dx$ $\quad = 4\,\frac{1}{2}\ln(x^2+9) + C = 2\ln(x^2+9) + C$

$\int \frac{4x}{x^2-9} dx$ $\quad = 2 \quad |x^2-9| + C$

$u = x^2 - 9$

$\int \frac{4}{x^2+9} dx$ $\quad = 4\int \frac{1}{x^2+9} dx = 4\int \frac{1}{9(\frac{x^2}{9}+1)} dx = \frac{4}{9}\int \frac{1}{(\frac{x}{3})^2+1} dx =$

$= \frac{4}{9}\, 3 \arctan(x/3) = \frac{4}{3}\arctan(x/3) + C$

$\int \frac{\ln(x)}{x} dx$

$= \frac{1}{2}(\ln(x))^2 + C$

$\int x e^x dx$ $\quad = xe^x - \int e^x dx = xe^x - e^x + C$

$f' = e^x \quad f = e^x$

$g = x \quad g' = 1$

$\int_0^\pi x^2 \sin(x) dx$ $\quad = -x^2\cos(x)\Big|_0^\pi + \int_0^\pi 2x\cos(x)dx = -x^2\cos(x)\Big|_0^\pi + 2x\sin(x)\Big|_0^\pi - 2\int_0^\pi \sin(x)dx$

$f' = \sin(x) \quad f = -\cos(x)$

$g = x^2 \quad g' = 2x$

$f' = \cos(x) \quad f = \sin(x)$

$g = 2x \quad g' = 2$

$= -\pi^2\cos(\pi) - 0 - 0 + 2\cos(x)\Big|_0^\pi$

$+ 2\cos(\pi) - 2\cos(0) =$

$= \pi^2 - 4$

$\int \ln(x) dx$ $\quad = x\ln(x) - x + C$

$\int 1 \cdot \ln$ $\quad$ by parts

$\int \cos^3(x)\sin^2(x)dx = \int \cos(x)\cdot\cos^2(x)\sin^2(x)\,dx = \int \cos(x)\,(1-\sin^2(x))\sin^2(x)\,dx =$

$u = \sin(x)$

$du = \cos(x)\,dx$

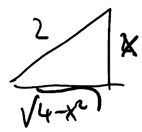
$= \int (1-u^2)u^2\,du = \int u^2 - u^4\,du =$

$= \frac{1}{3}(\sin^3(x)) - \frac{1}{5}\sin^5(x) + C$

$\int \frac{1}{x^2\sqrt{4-x^2}}dx = \int \frac{1}{4\sin^2(u)\,2\cancel{\cos(u)}}\,2\cancel{\cos(u)}\,du = \frac{1}{4}\int \csc^2(u)\,du = -\frac{1}{4}\cot(u) + C = -\frac{1}{4}\cot\left(\sin^{-1}\left(\frac{x}{2}\right)\right) + C$

$x = 2\sin(u)$

$dx = 2\cos(u)\,du$



$= -\frac{1}{4}\,\frac{\sqrt{4-x^2}}{x} + C$

$\int \frac{x-28}{x^2-x-6}$

$\frac{x-28}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2} \Rightarrow$

$x-28 = A(x+2) + B(x-3) \Rightarrow A = -5,\ B = 6$

$\int \frac{x-28}{x^2-x-6}dx = \int \frac{-5}{x-3}dx + \int \frac{6}{x+2}dx =$

$-5\ln(x-3) + 6\ln(x-2) + C$

$\int_0^{16} \frac{1}{\sqrt[4]{x}}dx = \lim_{t\to 0}\int_t^{16} x^{-1/4}dx = \lim_{t\to 0}\frac{4}{3}x^{3/4}\Big|_t^{16} = \lim_{t\to 0}\frac{4}{3}\left(16^{3/4} - t^{3/4}\right) = \frac{4}{3}\cdot 16^{3/4}$

$\int_0^\infty xe^{-x^2}dx = \lim_{t\to}\int_0^t xe^{-x^2}dx = \lim_{t\to\infty} -\frac{1}{2}e^{-x^2}\Big|_0^t = \lim_{t\to\infty} -\frac{1}{2}e^{-t^2} + 1/2 = \frac{1}{2}$

$\int_{-2}^{2} \frac{1}{\sqrt[3]{1-x}}dx = \begin{cases} = \int_{-2}^{1}(1-x)^{-1/3}dx + \int_1^2 (1-x)^{-1/3}dx = \lim_{t\to 1}\frac{3}{2}(1-x)^{2/3}\Big|_{-2}^{t} + \lim_{t\to 1}\frac{3}{2}(1-x)^{2/3}\Big|_t^2 \\ = \lim_{t\to 1}\frac{3}{2}(1-t)^{2/3} - \frac{3}{2}\cdot 3^{3/2} + \frac{3}{2}(-1)^{2/3} - \lim_{t\to 1}\frac{3}{2}(1-t)^{2/3} = -\frac{3}{2}\left(3^{3/2}+1\right) \end{cases}$

Please find the following limits (you might find l'Hospital's rule helpful for *some* limits)

$\lim_{x\to 0}\frac{\cos(x)}{x-1} = 1/{-1} = -1$

$$\lim_{x\to 0}\frac{\ln(x+1)}{x} = \lim_{x\to 0}\frac{1/x+1}{1} = 1/1 = 1$$

l'Hospital

$$\lim_{x\to\infty}\frac{x^2}{e^x} = \lim_{x\to\infty}\frac{2x}{e^x} = \lim_{x\to\infty}\frac{2}{e^x} = 0$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$ let $y = \left(1+\frac{1}{x}\right)^x \Rightarrow \ln(y) = x\ln\left(1+\frac{1}{x}\right) = \frac{\ln(1+\frac{1}{x})}{1/x}$

Then: $\lim_{x\to\infty}\frac{\ln(1+\frac{1}{x})}{1/x} = \lim_{x\to\infty}\frac{\frac{1}{1+1/x}\cdot\cancel{(-1/x^2)}}{\cancel{-1/x^2}} = 1 \Rightarrow \lim_{x\to\infty}\ln(y) = 1$ so that $\lim(y) = e^1 = e$

Find the arc length of the region bounded by the graph of $f(x)=\frac{2}{3}(x-1)^{3/2}$ where $0 \le x \le 1$

$L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx.$ $\quad f'(x) = (x-1)^{1/2} \Rightarrow 1+(f'(x))^2 = 1+x-1 = x$

$\Rightarrow L = \int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^1 = \frac{2}{3}$

Determine whether each of the following series (absolutely) converge or diverge. Please state carefully which test you are using to support your conclusion. If possible, find the limit of the series

a) $\sum_{n=1}^{\infty}\frac{n}{\ln(n)}$

$\lim\left(\frac{n}{\ln(n)}\right) = \infty \ne 0 \Rightarrow$ diverges by Div. Test

b) $\sum_{n=1}^{\infty}\frac{n-1}{n^3+n+1}$ $\quad \lim\frac{n-1}{n^3+n+1}\cdot\frac{n^2}{1} = 1 \Rightarrow$ limit comp. with $\sum 1/n^2$ so both converge

c) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (n+1) \cdot 3^n}{n!}$ $\lim \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{(n+2)\, 3^{n+1}}{(n+1)!} \cdot \frac{n!}{(n+1)\, 3^n} = \frac{(n+2) \cdot 3}{(n+1)(n+1)} = 0 < 1$

so converges by ratio test

d) $\sum_{n=5}^{\infty} \frac{3^n}{5^n}$ $= \sum_{n=5}^{\infty} \left(\frac{3}{5}\right)^n$ geometric series with $r = 3/5 \Rightarrow$ converges

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Recall that $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + ... + x^n + ... = \frac{1}{1-x}$ for $|x| < 1$. Use that fact to determine the power series centered at the origin for:

$f(x) = \frac{1}{1-4x^2}$ $= \sum_{n=0}^{\infty} (4x^2)^n = \sum_{n=0}^{\infty} 4^n x^{2n}$

$g(x) = -\ln(1-x)$ $= \int \frac{1}{1-x}\, dx = \int \sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} \frac{1}{n+1} x^{n+1}$

$h(x) = \frac{x^5}{1+x}$ $= x^5 \cdot \frac{1}{1-(-x)} = x^5 \cdot \sum (-x)^n = x^5 \sum (-1)^n x^n =$

$= \sum (-1)^n x^{n+5}$

Find the Taylor series for the following functions, all to be centered at the origin.

$x^3 e^{x^2}$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} \Rightarrow x^3 e^{x^2} = x^3 \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n+3}}{n!}$

$$\frac{\cos(x)-1}{x^2} = \frac{\left(\cancel{1} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots\right) \cancel{-1}}{x^2} = \frac{-\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots}{x^2}$$

$$= \frac{\cancel{x^2}\left(-\frac{1}{2!} + \frac{x^2}{4!} - \frac{x^4}{6!} + \frac{x^6}{8!} - \dots\right)}{\cancel{x^2}} = -\frac{1}{2!} + \frac{x^2}{4!} + \frac{x^4}{6!} - \dots = \underline{\underline{\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n-2}}{(2n)!}}}$$

$$\int e^{-x^2} dx$$

$$= \int \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} dx = \int \sum (-1)^n \frac{x^{2n}}{n!} dx = \sum_{n=0}^{\infty} (-1)^n \int \frac{x^{2n}}{n!} dx = \underline{\underline{\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!(2n+1)} x^{2n+1}}}$$

Suppose the indicated function has a power series *around 0*. Find the value of the specified term:

$f(x) = \sin(2x)\cos(3x)$, find a_1

$a_1 = \frac{f'(0)}{1!}$ so we need $f'(x)$ and plug in $x = 0$.

$f'(x) = 2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x) \Rightarrow f'(0) = 2 \Rightarrow \underline{\underline{a_1}} = 2/1! = \underline{\underline{2}}$

$f(x) = x - 3x^2 + x^3$, find a_{50}

$a_{50} = \frac{f^{(50)}(0)}{50!}$ so we need the 50^{th} derivative of $f(x)$.

But $f^{(50)}(x) = 0$, clearly, so that $\underline{\underline{a_{50}}} = 0/50! = \underline{\underline{0}}$

Which of the following functions are solutions to the indicated differential equations?

DE: $y' = xy$ - possible solution $y(x) = Ae^{\frac{x^2}{2}}$

$y' = Axe^{x^2/2}$

$= x \cdot Ae^{x^2/2} = xy$ so $\underline{\underline{\text{yes}}}$

DE: $y' = \frac{1}{2}(y^2 - 1)$ - possible solution $y(x) = \frac{1+ce^x}{1-ce^x}$

$y'(x) = \frac{ce^x(1-ce^x) + ce^x(1+ce^x)}{(1-ce^x)^2} = \frac{2ce^x}{(1-ce^x)^2}$

On the other hand $(y^2 - 1) = \frac{(1+ce^x)^2}{(1-ce^x)^2} - 1 = \frac{(1+ce^x)^2 - (1-ce^x)^2}{(1-ce^x)^2} =$

$= \frac{\cancel{1} + 2ce^x + \cancel{c^2e^{2x}} - \cancel{1} + 2ce^x - \cancel{c^2e^{2x}}}{(1-ce^x)^2} = \frac{4ce^x}{(1-ce^x)^2} \Rightarrow \frac{1}{2}(y^2-1) = y'$ so YES

DE: $y' + y\tan(x) = \cos^2(x)$ - possible solution $y(x) = \sin(x)\cos(x) - \cos(x)$

$y'(x) = \cos^2(x) - \sin^2(x) + \sin(x)$

$\Rightarrow y' + y\tan(x) = \cos^2(x) - \sin^2(x) + \sin(x) + (\sin(x)\cos(x) - \cos(x))\frac{\sin(x)}{\cos(x)} =$

$= \cos^2(x) - \cancel{\sin^2(x)} + \cancel{\sin(x)} + \cancel{\sin^2(x)} - \cancel{\sin(x)} = \cos^2(x)$ so YES

DE: $y' = xy^3$ with $y(0) = 2$ - possible solution $y(x) = (1-x^2)^{-\frac{1}{2}}$

Check initial condition first: $y(x) = (1-x^2)^{-1/2} \Rightarrow y(0) = 1 \neq 2$

so NO

The half-life of radium-266 is 1590 years. A sample of radium-226 has a mass of 100 mg. Find a formula for the mass of radium-226 after t years, using the law of radioactive decay.

a) Find the mass of the sample after 1000 years to the nearest milligram
b) When will the mass be reduced to 30 mg?

Model: $y = y_0 e^{rt} = 100e^{rt}$, know: $y(1590) = \boxed{50 = 100e^{k \cdot 1590}}$

$\Rightarrow \frac{1}{2} = e^{r \cdot 1590} \Rightarrow r = -\frac{1}{1590}\ln(2)$

a) $y(1000) = 100\, e^{-\ln(2)\frac{1000}{1590}} = 64.66$ mg (makes sense since after 1590 years there'd be 50 mg)

b) $y(t) = 30 = 100e^{-\ln(2)\cdot\frac{t}{1590}} \Rightarrow 0.3 = e^{-\ln(2)\frac{t}{1590}} \Rightarrow t = -\frac{\ln(0.3)}{\ln(2)} \cdot 1590 = 2761$ (makes sense)

Solve the following separable DE's

$$\frac{x\cos(x)}{y'} = 2y + e^{3y} \Leftrightarrow x\cos(x) = (2y + e^{3y})\frac{dy}{dx} \Leftrightarrow x\cos(x)\,dx = 2y + e^{3y}\,dy$$

$$\Leftrightarrow \int x\cos(x)\,dx = \int 2y + e^{3y}\,dy$$

$f' = \cos(x)$, $f = \sin(x)$
$g = x$, $g' = 1$

$$\Rightarrow \underline{x\sin(x) + \cos(x) + C = y^2 + \frac{1}{3}e^{3y}}$$

$xyy' = \ln(x)$ with $y(1) = 2$

$$xy\frac{dy}{dx} = \ln(x) \Leftrightarrow \int y\,dy = \int \frac{\ln(x)}{x}\,dx$$

$$\Leftrightarrow \frac{1}{2}y^2 = \frac{1}{2}(\ln(x))^2 + C \Rightarrow y^2 = (\ln(x))^2 + C$$

Since $(1,2)$ is part of the solution:

$$4 = 0 + C \Rightarrow \underline{C = 4} \Rightarrow \underline{y^2 = (\ln(x))^2 + 4}$$

$\frac{y'}{2x} = \sqrt{1 - y^2}$

$$\frac{dy}{\sqrt{1-y^2}} = 2x\,dx \Rightarrow \int \frac{1}{\sqrt{1-y^2}}\,dy = \int 2x\,dx \Leftrightarrow$$

$$\Leftrightarrow \arcsin(y) = x^2 + C \Rightarrow \underline{y = \sin(x^2 + C)}$$

$(2y^2 - 3y)y' = x\sin(x) \Leftrightarrow 2y^2 - 3y\,dy = x\sin(x)\,dx$

$$\int 2y^2 - 3y\,dy = \int x\sin(x)\,dx \Leftrightarrow \underline{\frac{2}{3}y^3 - \frac{3}{2}y = -x\cos(x) + \sin(x) + C}$$

$f' = \sin(x) \Rightarrow f = -\cos(x)$
$g = x$, $g' = 1$

Solve the following first order linear DE's.

$xy' = y + x^2\sin(x)$

$$y' - \frac{1}{x}y = x\sin(x) \Rightarrow \int p(x)\,dx = -\int \frac{1}{x}\,dx = -\ln(x) \Rightarrow u(x) = e^{-\ln(x)} = \underline{\frac{1}{x}}$$

Mult. by $u(x)$: $\frac{1}{x}y' - \frac{1}{x^2}y = \sin(x)$

or $\frac{d}{dx}\left(y \cdot \frac{1}{x}\right) = \sin(x) \Rightarrow y \cdot \frac{1}{x} = \int \sin(x)\,dx \Rightarrow y \cdot \frac{1}{x} = -\cos(x) + C$

$$\Rightarrow \underline{y = -x\cos(x) + Cx}$$

$$x\frac{dy}{dx} - \frac{y}{x+1} = x$$

$$\int P(x)\,dx = -\int \frac{1}{x(x+1)}\,dx = -\int \frac{1}{x} - \frac{1}{x+1}\,dx = \int \frac{1}{x+1} - \frac{1}{x}\,dx =$$

$$= \ln|x+1| - \ln|x| = \ln\left(\frac{x+1}{x}\right)$$

$$\Rightarrow u(x) = e^{\ln\left(\frac{x+1}{x}\right)} = \frac{x+1}{x}$$

$$y' - \frac{1}{x(x+1)}\,y = 1 \quad \Big|\ \frac{x+1}{x}$$

$$y'\left(\frac{x+1}{x}\right) - \frac{1}{x^2}\,y = \frac{x+1}{x}$$

or $$\frac{d}{dx}\left(y\cdot\frac{x+1}{x}\right) = \frac{x+1}{x} \Rightarrow y\,\frac{x+1}{x} = \int \frac{x+1}{x}\,dx = \int 1 + \frac{1}{x}\,dx = x + \ln|x| + C \Rightarrow \underline{y = \frac{x}{x+1}\left(x + \ln|x| + C\right)}$$

$$x^3y' - xy = 2xe^{-1/x}$$

$$y' - \frac{1}{x^2}\,y = \frac{2}{x^2}e^{-1/x} \Rightarrow \int P(x)\,dx = -\int \frac{1}{x^2}\,dx = \frac{1}{x} \Rightarrow u(x) = e^{1/x}$$

$$y'e^{1/x} - e^{1/x}\cdot\frac{1}{x^2}\,y = \frac{2}{x^2}e^{-1/x}e^{1/x}$$

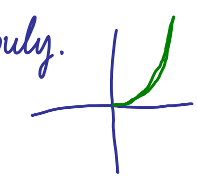
or $$\frac{d}{dx}\left(y e^{1/x}\right) = \frac{2}{x^2} \Rightarrow y e^{1/x} = \int \frac{2}{x^2}\,dx = -\frac{2}{x} + C \Rightarrow \underline{y = e^{-1/x}\left(C - \frac{2}{x}\right)}$$

Identify the following curves, given as parametric equations:

(t^2, t^6)

$x = t^2$, $y = t^6 = (t^2)^3 = (x)^3 \Rightarrow y = x^3$ but since $x = t^2$, x is positive only.

So: curve is $y = x^3$, $x \geq 0$



$x(t) = 2\cos(t)$, $y(t) = 2\sin(t)$

$x^2 + y^2 = 4\cos^2(t) + 4\sin^2(t) = 4\left(\cos^2(t) + \sin^2(t)\right) = 4 \Rightarrow x^2 + y^2 = 4$ so circle, center $(0,0)$ radius 4

$(2+3t, 1-2t)$

$x = 2 + 3t \Rightarrow x - 2 = 3t \Rightarrow \frac{1}{3}(x-2) = t$

$y = 1 - 2t = 1 - 2\cdot\frac{1}{3}(x-2) = 1 - \frac{2}{3}(x-2) = 1 - \frac{2}{3}x + \frac{4}{3} = \underline{-\frac{2}{3}x + 7/3}$

line slope $m = -2/3$ and y-intercept $7/3$

Find the parametric equation of a line through the points

(a) $(1,2)$ and ($(4,3)$

$(x, y) = (1,2) + t(4-1, 3-2) = (1,2) + t(3,1)$

$\Rightarrow (x, y) = (1+3t, 2+t)$ check: at $t=0$: $(x,y) = (1,2)$
if $t=1$: $(x,y) = (4,3)$ } line through 2 points, so check

b) $(2,-3)$ and $(2,1)$

$(x, y) = (2,-3) + t(2-2, 1-(-3)) = (2,-3) + t(0,4)$

$\Rightarrow (x, y) = (2, -3+4t)$ vertical line

} we did not cover this. Not on final!

For each of the parametric curves above, find

The derivative $(x'(t), y'(t))$

$(x, y) = (t^2, t^6) \Rightarrow (x', y') = (2t, 6t^5)$ (so not smooth at $t=0$)

$(x, y) = (2\cos(t), 2\sin(t)) \Rightarrow (x', y') = (-2\sin(t), 2\cos(t))$

$(x, y) = (2+3t, 1-2t) \Rightarrow (x', y') = (3, -2)$

The slope of the tangent line when $t = 1$

$(x, y) = (t^2, t^6) \Rightarrow \frac{dy}{dx} = y'/x' = \frac{6t^5}{2t} = 3t^4 \Rightarrow$ slope of tangent at $t=1$: $m = 3$

$(x, y) = (2\cos(t), 2\sin(t)) \Rightarrow \frac{dy}{dx} = \frac{y'}{x'} = \frac{2\cos(t)}{-2\sin(t)} = -\frac{\cos(t)}{\sin(t)} \Rightarrow$ slope at $t=1$: $m = -\frac{\cos(1)}{\sin(1)}$

$(x, y) = (2+3t, 1-2t) \Rightarrow \frac{dy}{dx} = \frac{y'}{x'} = \frac{-2}{3} \Rightarrow$ slope always: $m = -2/3$

The length of the curve for $0 \le t \le 1$ (skip the first curve)

$$L = \int_a^b \sqrt{(x')^2 + (y')^2}\,dt$$

$$(x,y) = (2\cos(t), 2\sin(t)) \Rightarrow L = \int_0^1 \sqrt{4\sin^2(t) + 4\cos^2(t)}\,dt = \int_0^1 2\,dt = \underline{\underline{2}}$$

$$(x,y) = (2+3t, 1-2t) \Rightarrow L = \int_0^1 \sqrt{9+4}\,dt = \sqrt{13}\cdot 1 = \underline{\underline{\sqrt{13}}}$$