Panel 1

Final:

Dec 18, Period 3 (12-2:30)

or

Dec 14, Period 4 (2:30)

Either option works!

Panel 2

From the Exam:

Solve $x y' + y = \sqrt{x}$ $\left(\frac{1}{x}\right)$

$y' + \frac{1}{x} y = \frac{1}{\sqrt{x}}$ $\quad | \cdot x$

$\int P(x)\,dx = \int \frac{1}{x}\,dx = \ln(x) \Rightarrow u(x) = e^{\ln(x)} = x$

$x y' + y = \sqrt{x}$

$\frac{d}{dx}(yx) = \sqrt{x} \Rightarrow yx = \int \sqrt{x}\,dx = \frac{2}{3} x^{3/2} + C \quad | \frac{1}{x}$

$$\boxed{y = \frac{2}{3} x^{1/2} + \frac{C}{x}}$$

Panel 3

Solve $\frac{x \cos(x)}{y'} = 3y^2$ with $y(0) = 2$ initial condition!

$x \cos(x) = 3y^2 \frac{dy}{dx}$

$x \cos(x)\,dx = 3y^2\,dy \quad | \int$

$\int x \cos(x)\,dx = \int 3y^2\,dy$

$f' = \cos(x) \quad f = \sin(x)$

$g = x \quad g' = 1$

$x \sin(x) - \int \sin(x)\,dx = y^3$

$$\boxed{x \sin(x) + \cos(x) + C = y^3}$$

$y = \sqrt[3]{x \sin(x) + \cos(x) + C}$

$y(0) = \sqrt[3]{1 + C} = 2 \Rightarrow C = 7$

$y = \sqrt[3]{x \sin(x) + \cos(x) + 7}$

Panel 4

Solve $y' = y^2 - x$ with $y(0) = 1$

Not sep., not linear

Solution as: $y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$, $a_n = \frac{y^{(n)}(0)}{n!}$

$y(0) = 1 \Rightarrow y(0) = a_0 = 1$ ✓

$a_1 = \frac{y'(0)}{1!}$ $\quad y' = y^2 - x \Rightarrow y'(0) = y(0)^2 - 0 = 1 \Rightarrow a_1 = 1$

$a_2 = \frac{y''(0)}{2!}$ $\quad y' = y^2 - x \quad | \frac{d}{dx}$

$y'' = 2y \cdot y' - 1 \Rightarrow y''(0) = 2y(0)y'(0) - 1 = 2 - 1 = 1 \Rightarrow a_2 = \frac{1}{2!}$

Panel 5

The story so far: DE: $y' = y^2 - x$

Taylor Series

Solution: $y(x) = y(0) + \frac{y'(0)}{1!}x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \dots$

and $y(0) = 1$, $y'(0) = 1$, $y''(0) = 1$ and $y'' = 2yy' - 1$ 1st der

$y'''(0)$: $y''' = 2y' \cdot y' + 2yy''$ $\Rightarrow$ $y'''(0) = 2(y'(0))^2 + 2y(0)y''(0) = 4$

Solution to DE with initial condition:

$$y(x) \sim 1 + 1x + \frac{1}{2}x^2 + \frac{4}{6}x^3 + \bigcirc x^4$$

Using Taylor series I can approx. DE solutions!

Panel 6

Ex: $y'' = -y$ $\qquad$ $y' = y \Rightarrow y = e^x$

$y(x) = \sin(x)$ or $y(x) = \cos(x)$ work because they work!

$y(x) = A\sin(x) + B\cos(x)$ is general solution!

Ex: $y'' + y' - 6y = 0$ $\qquad$ Guess: $y = e^{rx}$ or $y = \cancel{A}\sin(x)$, $y = \cancel{B}\cos(x)$

$y' = re^{rx}$, $y'' = r^2e^{rx}$

$r^2e^{rx} + re^{rx} - 6e^{rx} = 0$

$e^{rx}(r^2 + r - 6) = 0 \Rightarrow e^{rx}(r+3)(r-2) = 0$ $\quad$ $y = e^{-3x}$, $y = e^{2x}$

Panel 7

Ex: Show that the 2nd order DE $y'' + y' - 6y = 0$ has

$6\,y(x) = 6Ae^{2x} + 6Be^{-3x}$ as solution for all A, B

$y'(x) = 2Ae^{2x} - 3Be^{-3x}$

$y''(x) = 4Ae^{2x} + 9Be^{-3x}$

$y'' + y' - 6y = 0$

$y'' + ay' + by = 0$ is 2nd order homogeneous DE

solutions involve e^{rx}, $\sin(x)$, $\cos(x)$ $\qquad$ $e^{ix} = \cos(x) + i\sin(x)$ $\qquad$ here we Taylor Series

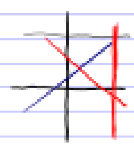
Complex Analysis

$\boxed{\text{Done with DE's!}}$

Panel 8

No more DE's - back to the very basics!

Lines

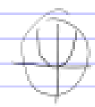


one line is different:

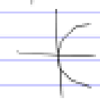
- it does not represent a function!
- it has ∞-slope

$y = mx + b$

Problem: $y = x^2$ is a function



$x = y^2$ is not a function!



should be treated the same!

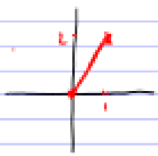
Panel 9

Def: $f(t)$, $g(t)$ continuous functions on $[a,b]$. Then the equations $x = f(t)$ and $y = g(t)$, or $(x(t), y(t)) = (f(t), g(t))$ is called parametric equation. The curve representing it is called parametric curve

flexible, useful,

Ex: $x = t$, $y = 2t$, $t \in [0,1]$. Sketch that:

t	x	y
0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	1
1	1	2



If $x = t$, $y = 2t$,

$y = 2x$

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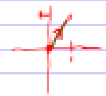
Panel 10

Ex: $(x,y) = (t, 2t)$, $t \in [0,1]$ describes the part of the line $y = 2x$ from $(0,0)$ to $(1,2)$

Ex: $(x,y) = (2t, 4t)$, $t \in [0, \frac{1}{2}]$ describes what curve?

$x = \underline{2t}$, $y = 4t = 2(\underline{2t}) = 2x$

same line segment!

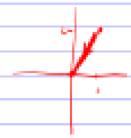


$(t, 2t)$, $t \in [0,1]$

Ex: $(x,y) = (1-t, 2-2t)$, $t \in [0,1]$ describes what curve?

$t = 0$: $(1,2)$

$t = 1$: $(0,0)$



again the same one!

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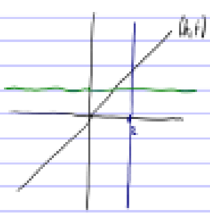
Panel 11

Note: A parametric curve $(x(t), y(t))$ has a direction and a speed that is usually not visible when you draw curve

Graph (t,t), $(2,t)$, $(t,1)$

$\boxed{(at+b,\ ct+d)}$ gives every line!



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Panel 12

Graph (t,t), $(2,t)$, and $(t,1)$

Done

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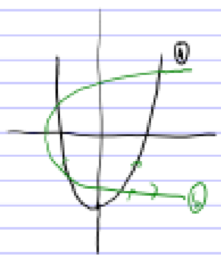
Panel 13

c) Graph $x = t/2$, $y = t^2 - 4$ $(\frac{t}{2}, t^2-4)$

d) and $x = t^2 - 4$, $y = t/2$ $(t^2-4, \frac{t}{2})$

a) $x = t/2 \Rightarrow 2x = t$, $y = t^2 - 4 \Rightarrow y = 4x^2 - 4$



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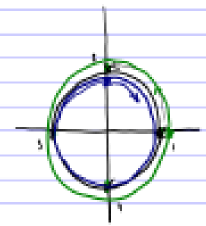
Panel 14

Sketch $(\cos(t), \sin(t))$, $(\sin(t), \cos(t))$, and $(\cos(2t), \sin(2t))$

t	x	y
0	1	0
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π		



$\cos^2(2t) + \sin^2(2t) = 1$

$x^2 + y^2 = 1$

$x = \cos(t)$, $y = \sin(t)$ $\Rightarrow x^2 + y^2 = 1$

green goes around twice as fast as black circle!

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Panel 15

To plot a parametric curve in Maple:

plot([x(t), y(t), t=a..b]) ↵

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