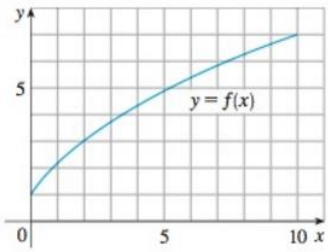
5.1 EXERCISES

1. (a) By reading values from the given graph of f, use five rectangles to find a lower estimate and an upper estimate for the area under the given graph of f from $x = 0$ to $x = 10$. In each case sketch the rectangles that you use.
(b) Find new estimates using ten rectangles in each case.

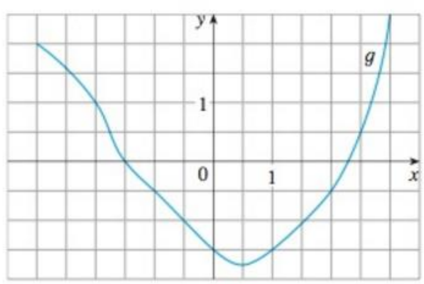


2. (a) Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 12$.
(i) L_6 (sample points are left endpoints)
(ii) R_6 (sample points are right endpoints)
(iii) M_6 (sample points are midpoints)
(b) Is L_6 an underestimate or overestimate of the true area?

4. (a) Estimate the area under the graph of $f(x) = 25 - x^2$ from $x = 0$ to $x = 5$ using five approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?
(b) Repeat part (a) using left endpoints.

5. (a) Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints. Then improve your estimate by using six rectangles. Sketch the curve and the approximating rectangles.
(b) Repeat part (a) using left endpoints.
(c) Repeat part (a) using midpoints.
(d) From your sketches in parts (a)–(c), which appears to be the best estimate?

6. (a) Graph the function $f(x) = e^{-x^2}$, $-2 \leqslant x \leqslant 2$.
(b) Estimate the area under the graph of f using four approximating rectangles and taking the sample points to be (i) right endpoints and (ii) midpoints. In each case sketch the curve and the rectangles.
(c) Improve your estimates in part (b) by using 8 rectangles.



9. A table of values of an increasing function f is shown. Use the table to find lower and upper estimates for $\int_0^{25} f(x)\,dx$.

x	0	5	10	15	20	25
$f(x)$	−42	−37	−25	−6	15	36

10. The table gives the values of a function obtained from an experiment. Use them to estimate $\int_0^6 f(x)\,dx$ using three equal subintervals with (a) right endpoints, (b) left end-

19–23 ▪ Use the form of the definition of the integral given in Theorem 4 to evaluate the integral.

19. $\int_{-1}^{5} (1 + 3x)\,dx$

20. $\int_{1}^{4} (x^2 + 2x - 5)\,dx$

21. $\int_{0}^{2} (2 - x^2)\,dx$

22. $\int_{0}^{5} (1 + 2x^3)\,dx$

23. $\int_{1}^{2} x^3\,dx$

24. (a) Find an approximation to the integral $\int_0^4 (x^2 - 3x)\,dx$ using a Riemann sum with right endpoints and $n = 8$.
(b) Draw a diagram like Figure 2 to illustrate the approximation in part (a).
(c) Use Theorem 4 to evaluate $\int_0^4 (x^2 - 3x)\,dx$.
(d) Interpret the integral in part (c) as a difference of areas and illustrate with a diagram like Figure 6.

25–26 ▪ Express the integral as a limit of Riemann sums. Do not evaluate the limit.

25. $\int_{2}^{6} \frac{x}{1 + x^5}\,dx$

26. $\int_{1}^{10} (x - 4 \ln x)\,dx$

31–36 ▪ Evaluate the integral by interpreting it in terms of areas.

31. $\int_{0}^{3} \left(\frac{1}{2}x - 1\right)dx$

32. $\int_{-2}^{2} \sqrt{4 - x^2}\,dx$

33. $\int_{-3}^{0} \left(1 + \sqrt{9 - x^2}\right)dx$

34. $\int_{-1}^{3} (3 - 2x)\,dx$

35. $\int_{-1}^{2} |x|\,dx$

36. $\int_{0}^{10} |x - 5|\,dx$

46. $\int_{0}^{2} \sqrt{x^3 + 1}\,dx$

47. $\int_{1}^{2} \frac{1}{x}\,dx$

48. $\int_{0}^{2} (x^3 - 3x + 3)\,dx$

49. $\int_{\pi/4}^{\pi/3} \tan x\,dx$

50. $\int_{\pi/4}^{3\pi/4} \sin^2 x\,dx$

51. Express the following limit as a definite integral:

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$$