

In Example 6 we used l'Hospital's Rule to show that

$$\lim_{x \rightarrow 0^+} x \ln x = 0$$

Therefore

$$\lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} e^{x \ln x} = e^0 = 1$$

5.8 EXERCISES

1–38 • Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

- $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - x}$
- $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$
- $\lim_{x \rightarrow (\pi/2)^+} \frac{\cos x}{1 - \sin x}$
- $\lim_{x \rightarrow 0} \frac{\sin 4x}{\tan 5x}$
- $\lim_{t \rightarrow 0} \frac{e^{2t} - 1}{\sin t}$
- $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}$
- $\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{1 + \cos 2\theta}$
- $\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{\csc \theta}$
- $\lim_{x \rightarrow 0^+} \frac{\ln x}{x}$
- $\lim_{x \rightarrow 0} \frac{\ln \sqrt{x}}{x^2}$
- $\lim_{t \rightarrow 1} \frac{t^8 - 1}{t^5 - 1}$
- $\lim_{t \rightarrow 0} \frac{8^t - 5^t}{t}$
- $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$
- $\lim_{u \rightarrow \infty} \frac{e^{u/10}}{u^3}$
- $\lim_{x \rightarrow 0} \frac{x3^x}{3^x - 1}$
- $\lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2}$
- $\lim_{x \rightarrow 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$
- $\lim_{x \rightarrow 0} \frac{x}{\tan^{-1}(4x)}$
- $\lim_{x \rightarrow 1} \frac{x^a - ax + a - 1}{(x - 1)^2}$
- $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$
- $\lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$
- $\lim_{x \rightarrow a} \frac{\cos x \ln(x - a)}{\ln(e^x - e^a)}$
- $\lim_{x \rightarrow 0} \cot 2x \sin 6x$
- $\lim_{x \rightarrow \infty} \sqrt{x} e^{-x/2}$
- $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$
- $\lim_{x \rightarrow 0} \sin x \ln x$
- $\lim_{x \rightarrow 1^+} \ln x \tan(\pi x/2)$
- $\lim_{x \rightarrow \infty} x \tan(1/x)$
- $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$
- $\lim_{x \rightarrow 0} (\csc x - \cot x)$
- $\lim_{x \rightarrow \infty} (x - \ln x)$
- $\lim_{x \rightarrow 1^-} [\ln(x^2 - 1) - \ln(x^5 - 1)]$

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- $\lim_{x \rightarrow 0^+} x^{\sqrt{x}}$
- $\lim_{x \rightarrow 0^+} (\tan 2x)^x$
- $\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$
- $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} \right)^{bx}$
- $\lim_{x \rightarrow 1^+} x^{1/(1-x)}$
- $\lim_{x \rightarrow \infty} (e^x + x)^{1/x}$

39–40 • Use a graph to estimate the value of the limit. Then use l'Hospital's Rule to find the exact value.

- $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x} \right)^x$
- $\lim_{x \rightarrow 0} \frac{5^x - 4^x}{3^x - 2^x}$

41. Prove that

$$\lim_{x \rightarrow \infty} \frac{e^x}{x^n} = \infty$$

for any positive integer n . This shows that the exponential function approaches infinity faster than any power of x .

42. Prove that

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^p} = 0$$

for any number $p > 0$. This shows that the logarithmic function approaches ∞ more slowly than any power of x .

43–44 • What happens if you try to use l'Hospital's Rule to find the limit? Evaluate the limit using another method.

- $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}}$
- $\lim_{x \rightarrow (\pi/2)^-} \frac{\sec x}{\tan x}$

45. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is

$$A = A_0 \left(1 + \frac{r}{n} \right)^{nt}$$

If we let $n \rightarrow \infty$, we refer to the *continuous compounding* of interest. Use l'Hospital's Rule to show that if interest is compounded continuously, then the amount after t years is

$$A = A_0 e^{rt}$$