In Example 6 we used l'Hospital's Rule to show that

$$\lim_{x\to 0^+} x \ln x = 0$$

Therefore $$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x} = e^0 = 1$$ ■

5.8 EXERCISES

1–38 ▪ Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

1. $\lim_{x\to 1} \dfrac{x^2-1}{x^2-x}$

2. $\lim_{x\to 2} \dfrac{x^2+x-6}{x-2}$

3. $\lim_{x\to(\pi/2)^+} \dfrac{\cos x}{1-\sin x}$

4. $\lim_{x\to 0} \dfrac{\sin 4x}{\tan 5x}$

5. $\lim_{t\to 0} \dfrac{e^{2t}-1}{\sin t}$

6. $\lim_{x\to 0} \dfrac{x^2}{1-\cos x}$

7. $\lim_{\theta\to\pi/2} \dfrac{1-\sin\theta}{1+\cos 2\theta}$

8. $\lim_{\theta\to\pi/2} \dfrac{1-\sin\theta}{\csc\theta}$

9. $\lim_{x\to 0^+} \dfrac{\ln x}{x}$

10. $\lim_{x\to\infty} \dfrac{\ln\sqrt{x}}{x^2}$

11. $\lim_{t\to 1} \dfrac{t^8-1}{t^5-1}$

12. $\lim_{t\to 0} \dfrac{8^t-5^t}{t}$

13. $\lim_{x\to 0} \dfrac{e^x-1-x}{x^2}$

14. $\lim_{u\to\infty} \dfrac{e^{u/10}}{u^3}$

15. $\lim_{x\to 0} \dfrac{x3^x}{3^x-1}$

16. $\lim_{x\to 0} \dfrac{\cos mx-\cos nx}{x^2}$

17. $\lim_{x\to 1} \dfrac{1-x+\ln x}{1+\cos\pi x}$

18. $\lim_{x\to 0} \dfrac{x}{\tan^{-1}(4x)}$

19. $\lim_{x\to 1} \dfrac{x^a-ax+a-1}{(x-1)^2}$

20. $\lim_{x\to 0} \dfrac{e^x-e^{-x}-2x}{x-\sin x}$

21. $\lim_{x\to 0} \dfrac{\cos x-1+\frac{1}{2}x^2}{x^4}$

22. $\lim_{x\to a^+} \dfrac{\cos x\ln(x-a)}{\ln(e^x-e^a)}$

23. $\lim_{x\to 0} \cot 2x \sin 6x$

24. $\lim_{x\to\infty} \sqrt{x}\, e^{-x/2}$

25. $\lim_{x\to\infty} x^3 e^{-x^2}$

26. $\lim_{x\to 0^+} \sin x \ln x$

27. $\lim_{x\to 1^+} \ln x \tan(\pi x/2)$

28. $\lim_{x\to\infty} x\tan(1/x)$

29. $\lim_{x\to 0^+} \left(\dfrac{1}{x}-\dfrac{1}{e^x-1}\right)$

30. $\lim_{x\to 0} (\csc x-\cot x)$

31. $\lim_{x\to\infty} (x-\ln x)$

32. $\lim_{x\to 1^+} [\ln(x^7-1)-\ln(x^5-1)]$

33. $\lim_{x\to 0^+} x^{\sqrt{x}}$

34. $\lim_{x\to 0^+} (\tan 2x)^x$

35. $\lim_{x\to 0} (1-2x)^{1/x}$

36. $\lim_{x\to\infty} \left(1+\dfrac{a}{x}\right)^{bx}$

37. $\lim_{x\to 1^+} x^{1/(1-x)}$

38. $\lim_{x\to\infty} (e^x+x)^{1/x}$

39–40 ▪ Use a graph to estimate the value of the limit. Then use l'Hospital's Rule to find the exact value.

39. $\lim_{x\to\infty} \left(1+\dfrac{2}{x}\right)^x$

40. $\lim_{x\to 0} \dfrac{5^x-4^x}{3^x-2^x}$

41. Prove that

$$\lim_{x\to\infty} \frac{e^x}{x^n} = \infty$$

for any positive integer n. This shows that the exponential function approaches infinity faster than any power of x.

42. Prove that

$$\lim_{x\to\infty} \frac{\ln x}{x^p} = 0$$

for any number $p > 0$. This shows that the logarithmic function approaches ∞ more slowly than any power of x.

43–44 ▪ What happens if you try to use l'Hospital's Rule to find the limit? Evaluate the limit using another method.

43. $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$

44. $\lim_{x\to(\pi/2)^-} \dfrac{\sec x}{\tan x}$

45. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is

$$A = A_0\left(1+\frac{r}{n}\right)^{nt}$$

If we let $n\to\infty$, we refer to the *continuous compounding* of interest. Use l'Hospital's Rule to show that if interest is compounded continuously, then the amount after t years is

$$A = A_0 e^{rt}$$