

166 CHAPTER 3 INVERSE FUNCTIONS

To differentiate a function of the form $y = [f(x)]^{g(x)}$, where both the base and the exponent are functions, logarithmic differentiation can be used as in the following example.

EXAMPLE 11 Differentiate $y = x^{\sqrt{x}}$.

SOLUTION 1 Using logarithmic differentiation, we have

$$\begin{aligned} \ln y &= \ln x^{\sqrt{x}} = \sqrt{x} \ln x \\ \frac{y'}{y} &= \sqrt{x} \cdot \frac{1}{x} + (\ln x) \frac{1}{2\sqrt{x}} \\ y' &= y \left(\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}} \right) = x^{\sqrt{x}} \left(\frac{2 + \ln x}{2\sqrt{x}} \right) \end{aligned}$$

SOLUTION 2 Another method is to write $x^{\sqrt{x}} = (e^{\ln x})^{\sqrt{x}}$:

$$\begin{aligned} \frac{d}{dx}(x^{\sqrt{x}}) &= \frac{d}{dx}(e^{\sqrt{x} \ln x}) = e^{\sqrt{x} \ln x} \frac{d}{dx}(\sqrt{x} \ln x) \\ &= x^{\sqrt{x}} \left(\frac{2 + \ln x}{2\sqrt{x}} \right) \quad (\text{as in Solution 1}) \end{aligned}$$

Figure 4 illustrates Example 11 by showing the graphs of $f(x) = x^{\sqrt{x}}$ and its derivative.

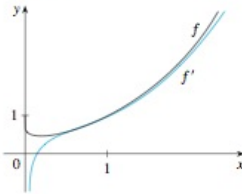


FIGURE 4

3.3 EXERCISES

1–36 • Differentiate the function.

1. $f(x) = \log_2(1 - 3x)$
2. $f(x) = \ln(x^2 + 10)$
3. $f(\theta) = \ln(\cos \theta)$
4. $f(x) = \cos(\ln x)$
5. $f(x) = \sqrt[3]{\ln x}$
6. $f(x) = \ln \sqrt[3]{x}$
7. $f(x) = \sin x \ln(5x)$
8. $f(x) = \log_5(xe^x)$
9. $g(x) = \ln \frac{a-x}{a+x}$
10. $f(t) = \frac{1 + \ln t}{1 - \ln t}$
11. $F(t) = \ln \frac{(2t+1)^3}{(3t-1)^4}$
12. $f(x) = \log_{10} \left(\frac{x}{x-1} \right)$
13. $f(u) = \frac{\ln u}{1 + \ln(2u)}$
14. $y = \ln(x^4 \sin^2 x)$
15. $y = \ln |2 - x - 5x^2|$
16. $G(u) = \ln \sqrt{\frac{3u+2}{3u-2}}$
17. $f(x) = x^2 e^x$
18. $g(x) = \sqrt{x} e^x$
19. $y = \frac{e^x}{x^2}$
20. $y = \frac{e^x}{1+x}$
21. $y = xe^{-x^2}$
22. $y = e^{-5x} \cos 3x$
23. $y = e^{x \cos x}$
24. $y = 10^{1-x^2}$
25. $h(t) = t^3 - 3^t$
26. $y = \frac{1}{s + ke^t}$

27. $y = \frac{ae^t + b}{ce^t + d}$
28. $f(x) = \frac{1 - xe^x}{x + e^x}$
29. $y = 2^{\sin x}$
30. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
31. $f(u) = e^{1/u}$
32. $y = e^{x \tan \sqrt{x}}$
33. $y = \ln(e^{-x} + xe^{-x})$
34. $y = [\ln(1 + e^x)]^2$
35. $F(t) = e^{t \sin 2t}$
36. $y = 2^{3^t}$
- 37–40 • Find y' and y'' .
37. $y = e^{\sin x} \sin \beta x$
38. $y = \frac{\ln x}{x^2}$
39. $y = x \ln x$
40. $y = \ln(\sec x + \tan x)$
- 41–42 • Find an equation of the tangent line to the curve at the given point.
41. $y = \ln \ln x$, $(e, 0)$
42. $y = e^x/x$, $(1, e)$
- 43–44 • Differentiate f and find the domain of f .
43. $f(x) = \frac{x}{1 - \ln(x-1)}$
44. $f(x) = \ln \ln x$