

## 2.6 EXERCISES

## 1–2 •

- (a) Find  $y'$  by implicit differentiation.  
 (b) Solve the equation explicitly for  $y$  and differentiate to get  $y'$  in terms of  $x$ .  
 (c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for  $y$  into your solution for part (a).

1.  $9x^2 - y^2 = 1$                       2.  $2x^2 + x + xy = 1$

3–16 • Find  $dy/dx$  by implicit differentiation.

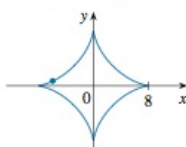
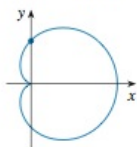
3.  $x^3 + y^3 = 1$                       4.  $2x^3 + x^2y - xy^3 = 2$   
 5.  $x^2 + xy - y^2 = 4$               6.  $y^5 + x^2y^3 = 1 + x^4y$   
 7.  $y \cos x = x^2 + y^2$               8.  $\cos(xy) = 1 + \sin y$   
 9.  $4 \cos x \sin y = 1$                 10.  $y \sin(x^2) = x \sin(y^2)$   
 11.  $\tan(x/y) = x + y$               12.  $\sqrt{x+y} = 1 + x^2y^2$   
 13.  $\sqrt{xy} = 1 + x^2y$                 14.  $x \sin y + y \sin x = 1$   
 15.  $y \cos x = 1 + \sin(xy)$         16.  $\tan(x - y) = \frac{y}{1 + x^2}$

17. If  $f(x) + x^2[f(x)]^3 = 10$  and  $f(1) = 2$ , find  $f'(1)$ .

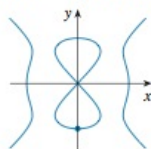
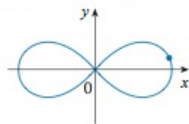
18. If  $g(x) + x \sin g(x) = x^2$ , find  $g'(0)$ .

## 19–24 • Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

19.  $x^2 + xy + y^2 = 3$ ,  $(1, 1)$  (ellipse)  
 20.  $x^2 + 2xy - y^2 + x = 2$ ,  $(1, 2)$  (hyperbola)  
 21.  $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$     22.  $x^{2/3} + y^{2/3} = 4$   
      $(0, \frac{1}{2})$  (cardioid)                       $(-3\sqrt{3}, 1)$  (astroid)



23.  $2(x^2 + y^2)^2 = 25(x^2 - y^2)$     24.  $y^2(y^2 - 4) = x^2(x^2 - 5)$   
      $(3, 1)$  (lemniscate)                       $(0, -2)$  (devil's curve)



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25–28 • Find  $y''$  by implicit differentiation.

25.  $9x^2 + y^2 = 9$                       26.  $\sqrt{x} + \sqrt{y} = 1$

27.  $x^3 + y^3 = 1$                         28.  $x^4 + y^4 = a^4$

29. (a) The curve with equation  $y^2 = 5x^4 - x^2$  is called a **kampyle of Eudoxus**. Find an equation of the tangent line to this curve at the point  $(1, 2)$ .

(b) Illustrate part (a) by graphing the curve and the tangent line on a common screen. (If your graphing device will graph implicitly defined curves, then use that capability. If not, you can still graph this curve by graphing its upper and lower halves separately.)

30. (a) The curve with equation  $y^2 = x^3 + 3x^2$  is called the **Tschirnhausen cubic**. Find an equation of the tangent line to this curve at the point  $(1, -2)$ .

(b) At what points does this curve have a horizontal tangent?

(c) Illustrate parts (a) and (b) by graphing the curve and the tangent lines on a common screen.

## CAS 31. Fanciful shapes can be created by using the implicit plotting capabilities of computer algebra systems.

- (a) Graph the curve with equation

$$y(y^2 - 1)(y - 2) = x(x - 1)(x - 2)$$

At how many points does this curve have horizontal tangents? Estimate the  $x$ -coordinates of these points.

- (b) Find equations of the tangent lines at the points  $(0, 1)$  and  $(0, 2)$ .  
 (c) Find the exact  $x$ -coordinates of the points in part (a).  
 (d) Create even more fanciful curves by modifying the equation in part (a).

## CAS 32. (a) The curve with equation

$$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$$

has been likened to a bouncing wagon. Use a computer algebra system to graph this curve and discover why.

- (b) At how many points does this curve have horizontal tangent lines? Find the  $x$ -coordinates of these points.

33. Find the points on the lemniscate in Exercise 23 where the tangent is horizontal.

34. Show by implicit differentiation that the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point  $(x_0, y_0)$  is

$$\frac{x_0x}{a^2} + \frac{y_0y}{b^2} = 1$$

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**35–38** ■ Two curves are **orthogonal** if their tangent lines are perpendicular at each point of intersection. Show that the given families of curves are **orthogonal trajectories** of each other, that is, every curve in one family is orthogonal to every curve in the other family. Sketch both families of curves on the same axes.

35.  $x^2 + y^2 = r^2$ ,  $ax + by = 0$

36.  $x^2 + y^2 = ax$ ,  $x^2 + y^2 = by$

37.  $y = cx^2$ ,  $x^2 + 2y^2 = k$

38.  $y = ax^3$ ,  $x^2 + 3y^2 = b$

39. (a) The *van der Waals equation* for  $n$  moles of a gas is

$$\left(P + \frac{n^2a}{V^2}\right)(V - nb) = nRT$$

where  $P$  is the pressure,  $V$  is the volume, and  $T$  is the temperature of the gas. The constant  $R$  is the universal gas constant and  $a$  and  $b$  are positive constants that are characteristic of a particular gas. If  $T$  remains constant, use implicit differentiation to find  $dV/dP$ .

(b) Find the rate of change of volume with respect to pressure of 1 mole of carbon dioxide at a volume of  $V = 10$  L and a pressure of  $P = 2.5$  atm. Use  $a = 3.592$  L<sup>2</sup>·atm/mole<sup>2</sup> and  $b = 0.04267$  L/mole.

40. (a) Use implicit differentiation to find  $y'$  if  $x^2 + xy + y^2 + 1 = 0$ .

**CAS** (b) Plot the curve in part (a). What do you see? Prove that what you see is correct.

(c) In view of part (b), what can you say about the expression for  $y'$  that you found in part (a)?

41. Show, using implicit differentiation, that any tangent line at a point  $P$  to a circle with center  $O$  is perpendicular to the radius  $OP$ .

42. Show that the sum of the  $x$ - and  $y$ -intercepts of any tangent line to the curve  $\sqrt{x} + \sqrt{y} = \sqrt{c}$  is equal to  $c$ .

43. The equation  $x^2 - xy + y^2 = 3$  represents a “rotated ellipse,” that is, an ellipse whose axes are not parallel to

the coordinate axes. Find the points at which this ellipse crosses the  $x$ -axis and show that the tangent lines at these points are parallel.

44. (a) Where does the normal line to the ellipse  $x^2 - xy + y^2 = 3$  at the point  $(-1, 1)$  intersect the ellipse a second time?

**✎** (b) Illustrate part (a) by graphing the ellipse and the normal line.

45. Find all points on the curve  $x^2y^2 + xy = 2$  where the slope of the tangent line is  $-1$ .

46. Find equations of both the tangent lines to the ellipse  $x^2 + 4y^2 = 36$  that pass through the point  $(12, 3)$ .

47. Show that the ellipse  $x^2/a^2 + y^2/b^2 = 1$  and the hyperbola  $x^2/A^2 - y^2/B^2 = 1$  are orthogonal trajectories if  $A^2 < a^2$  and  $a^2 - b^2 = A^2 + B^2$  (so the ellipse and hyperbola have the same foci).

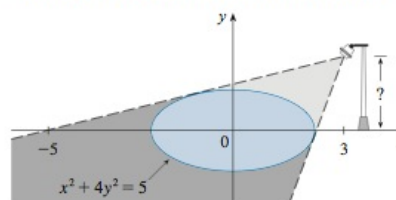
48. Find the value of the number  $a$  such that the families of curves  $y = (x + c)^{-1}$  and  $y = a(x + k)^{1/3}$  are orthogonal trajectories.

49. The **Bessel function** of order 0,  $y = J(x)$ , satisfies the differential equation  $xy'' + y' + xy = 0$  for all values of  $x$  and its value at 0 is  $J(0) = 1$ .

(a) Find  $J'(0)$ .

(b) Use implicit differentiation to find  $J''(0)$ .

50. The figure shows a lamp located three units to the right of the  $y$ -axis and a shadow created by the elliptical region  $x^2 + 4y^2 \leq 5$ . If the point  $(-5, 0)$  is on the edge of the shadow, how far above the  $x$ -axis is the lamp located?



## 2.7 RELATED RATES

If we are pumping air into a balloon, both the volume and the radius of the balloon are increasing and their rates of increase are related to each other. But it is much easier to measure directly the rate of increase of the volume than the rate of increase of the radius.

In a related rates problem the idea is to compute the rate of change of one quantity in terms of the rate of change of another quantity (which may be more easily mea-

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