

3.5 EXERCISES

1. Consider the following problem: Find two numbers whose sum is 23 and whose product is a maximum.

(a) Make a table of values, like the following one, so that the sum of the numbers in the first two columns is always 23. On the basis of the evidence in your table, estimate the answer to the problem.

First number	Second number	Product
1	22	22
2	21	42
3	20	60
.	.	.
.	.	.

- (b) Use calculus to solve the problem and compare with your answer to part (a).
2. Find two numbers whose difference is 100 and whose product is a minimum.
3. Find two positive numbers whose product is 100 and whose sum is a minimum.
4. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?
5. What is the maximum vertical distance between the line $y = x + 2$ and the parabola $y = x^2$ for $-1 \leq x \leq 2$?
6. What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?
7. Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.
8. Find the dimensions of a rectangle with area 1000 m^2 whose perimeter is as small as possible.
9. Consider the following problem: A farmer with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?
- (a) Draw several diagrams illustrating the situation, some with shallow, wide pens and some with deep, narrow pens. Find the total areas of these configurations. Does it appear that there is a maximum area? If so, estimate it.
- (b) Draw a diagram illustrating the general situation. Introduce notation and label the diagram with your symbols.
- (c) Write an expression for the total area.
- (d) Use the given information to write an equation that relates the variables.
- (e) Use part (d) to write the total area as a function of one variable.
- (f) Finish solving the problem and compare the answer with your estimate in part (a).

10. Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

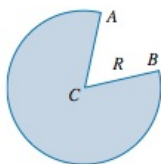
- (a) Draw several diagrams to illustrate the situation, some short boxes with large bases and some tall boxes with small bases. Find the volumes of several such boxes. Does it appear that there is a maximum volume? If so, estimate it.
- (b) Draw a diagram illustrating the general situation. Introduce notation and label the diagram with your symbols.
- (c) Write an expression for the volume.
- (d) Use the given information to write an equation that relates the variables.
- (e) Use part (d) to write the volume as a function of one variable.
- (f) Finish solving the problem and compare the answer with your estimate in part (a).

11. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.
12. A box with a square base and open top must have a volume of $32,000 \text{ cm}^3$. Find the dimensions of the box that minimize the amount of material used.
13. (a) Show that of all the rectangles with a given area, the one with smallest perimeter is a square.
(b) Show that of all the rectangles with a given perimeter, the one with greatest area is a square.
14. A rectangular storage container with an open top is to have a volume of 10 m^3 . The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.
15. Find the point on the line $y = 2x + 3$ that is closest to the origin.
16. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.
17. Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.
18. Find, correct to two decimal places, the coordinates of the point on the curve $y = \sin x$ that is closest to the point $(4, 2)$.
19. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.
20. Find the area of the largest trapezoid that can be inscribed in a circle of radius 1 and whose base is a diameter of the circle.

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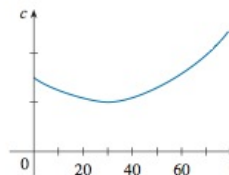
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21. Find the dimensions of the isosceles triangle of largest area that can be inscribed in a circle of radius r .
22. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs.
23. A right circular cylinder is inscribed in a sphere of radius r . Find the largest possible volume of such a cylinder.
24. Find the area of the largest rectangle that can be inscribed in the ellipse $x^2/a^2 + y^2/b^2 = 1$.
25. A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.
26. A right circular cylinder is inscribed in a cone with height h and base radius r . Find the largest possible volume of such a cylinder.
27. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) A minimum?
28. A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?
29. A cone-shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining the edges CA and CB . Find the maximum capacity of such a cup.



30. A cone-shaped paper drinking cup is to be made to hold 27 cm^3 of water. Find the height and radius of the cup that will use the smallest amount of paper.
31. A cone with height h is inscribed in a larger cone with height H so that its vertex is at the center of the base of the larger cone. Show that the inner cone has maximum volume when $h = \frac{1}{3}H$.
32. The graph shows the fuel consumption c of a car (measured in gallons per hour) as a function of the speed v of the car. At very low speeds the engine runs inefficiently, so initially c decreases as the speed increases. But at high speeds the fuel consumption increases. You can see that $c(v)$ is minimized for this car when $v \approx 30 \text{ mi/h}$. However, for fuel

efficiency, what must be minimized is not the consumption in gallons per hour but rather the fuel consumption in gallons *per mile*. Let's call this consumption G . Using the graph, estimate the speed at which G has its minimum value.



33. If a resistor of R ohms is connected across a battery of E volts with internal resistance r ohms, then the power (in watts) in the external resistor is

$$P = \frac{E^2 R}{(R + r)^2}$$

If E and r are fixed but R varies, what is the maximum value of the power?

34. For a fish swimming at a speed v relative to the water, the energy expenditure per unit time is proportional to v^3 . It is believed that migrating fish try to minimize the total energy required to swim a fixed distance. If the fish are swimming against a current u ($u < v$), then the time required to swim a distance L is $L/(v - u)$ and the total energy E required to swim the distance is given by

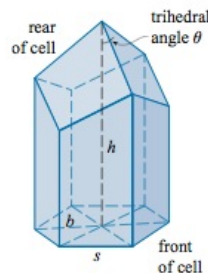
$$E(v) = av^3 \cdot \frac{L}{v - u}$$

where a is the proportionality constant.

- (a) Determine the value of v that minimizes E .
- (b) Sketch the graph of E .

Note: This result has been verified experimentally; migrating fish swim against a current at a speed 50% greater than the current speed.

35. In a beehive, each cell is a regular hexagonal prism, open at one end with a trihedral angle at the other end as in the figure. It is believed that bees form their cells in such a way



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as to minimize the surface area for a given side length and height, thus using the least amount of wax in cell construction. Examination of these cells has shown that the measure of the apex angle θ is amazingly consistent. Based on the geometry of the cell, it can be shown that the surface area S is given by

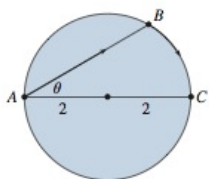
$$S = 6sh - \frac{3}{2}s^2 \cot \theta + (3s^2\sqrt{3}/2) \csc \theta$$

where s , the length of the sides of the hexagon, and h , the height, are constants.

- Calculate $dS/d\theta$.
- What angle should the bees prefer?
- Determine the minimum surface area of the cell (in terms of s and h).

Note: Actual measurements of the angle θ in beehives have been made, and the measures of these angles seldom differ from the calculated value by more than 2° .

- A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?
- The illumination of an object by a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. If two light sources, one three times as strong as the other, are placed 10 ft apart, where should an object be placed on the line between the sources so as to receive the least illumination?
- A woman at a point A on the shore of a circular lake with radius 2 mi wants to arrive at the point C diametrically opposite A on the other side of the lake in the shortest possible time. She can walk at the rate of 4 mi/h and row a boat at 2 mi/h. How should she proceed?



- Find an equation of the line through the point $(3, 5)$ that cuts off the least area from the first quadrant.
- At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?
- What is the shortest possible length of the line segment that is cut off by the first quadrant and is tangent to the curve $y = 3/x$ at some point?
- What is the smallest possible area of the triangle that is cut off by the first quadrant and whose hypotenuse is tangent to the parabola $y = 4 - x^2$ at some point?

- If $C(x)$ is the cost of producing x units of a commodity, then the **average cost** per unit is $c(x) = C(x)/x$. Show that if the average cost is a minimum, then the marginal cost equals the average cost.
 - If $C(x) = 16,000 + 200x + 4x^{3/2}$, in dollars, find (i) the cost, average cost, and marginal cost at a production level of 1000 units; (ii) the production level that will minimize the average cost; and (iii) the minimum average cost.
 - Show that if the profit $P(x)$ is a maximum, then the marginal revenue equals the marginal cost.
 - If $C(x) = 16,000 + 500x - 1.6x^2 + 0.004x^3$ is the cost function and $p(x) = 1700 - 7x$ is the demand function, find the production level that will maximize profit.
 - A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at \$10, the average attendance had been 27,000. When ticket prices were lowered to \$8, the average attendance rose to 33,000.
 - Find the demand function, assuming that it is linear.
 - How should ticket prices be set to maximize revenue?
 - During the summer months Terry makes and sells necklaces on the beach. Last summer he sold the necklaces for \$10 each and his sales averaged 20 per day. When he increased the price by \$1, he found that the average decreased by two sales per day.
 - Find the demand function, assuming that it is linear.
 - If the material for each necklace costs Terry \$6, what should the selling price be to maximize his profit?
 - A manufacturer has been selling 1000 flat-screen TVs a week at \$450 each. A market survey indicates that for each \$10 rebate offered to the buyer, the number of TVs sold will increase by 100 per week.
 - Find the demand function.
 - How large a rebate should the company offer the buyer in order to maximize its revenue?
 - If its weekly cost function is $C(x) = 68,000 + 150x$, how should the manufacturer set the size of the rebate in order to maximize its profit?
 - The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager charge to maximize revenue?
 - Let a and b be positive numbers. Find the length of the shortest line segment that is cut off by the first quadrant and passes through the point (a, b) .
- CAS 50.** The frame for a kite is to be made from six pieces of wood. The four exterior pieces have been cut with the lengths

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