**Finding local Extrema and Increasing/Decreasing Intervals**

Recall that a local extrema is either a local minimum or a local maximum. We already know that if a function has a local extremum at and it is differentiable at , then . Thus, to find out local max/mins we can follow the following **recipe**:

1. Compute the derivative
2. Solve as well as where does not exist. These points are called critical points or potential extrema
3. Create a table with and in the rows and the critical points defining the columns. Determine the sign of in each column and write down what that means for : if is positive, is increasing (goes up), if is negative, is decreasing (goes down).

Read off the local extrema and write down your answers.

**Example**: Find all relative extrema for

1. so that and are critical points
2. Setting up the table:

|  |  |  |  |
| --- | --- | --- | --- |
|  | -1 0 |  ½ 1 |  2 |
|  | + | - | + |
|  |  |  |  |

To determine the signs of the derivative, I used ‘test numbers’ in the various intervals, substituted them into the derivative (factored version) and figured out the sign only. It is easy, since I only need to figure out the signs of the various factors, from which it is easy to find the overall sign.

Now is a relative max, while is a relative min.

**Example**: Find all relative extrema for

1. so that , , and are critical points
2. Setting up the table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  -6 - |  -1 0 | 1  | 6 |
|  | + | - | - | + |
|  |  |  |  |  |



To determine the signs of the derivative, I again used ‘test numbers’ in the various intervals.

Now is a relative max, while is a relative min. The critical point turns out to be neither max nor min (see graph on the right).

**Finding Inflection Points and Concavity**

Recall that is an inflection point if the concavity changes at that point. Since the second derivative tells us whether the function is concave up (smile face, ) or concave down (frowny face, ), we follow the following **recipe** to discuss concavity:

1. Compute the second derivative
2. Solve as well as where does not exist. These points are called possible inflection points.
3. Create a table with and in the rows and the possible inflection points defining the columns. Determine the sign of in each column and write down what that means for . Note: if is positive, is concave up, if is negative, is concave down.

**Example**: Discuss the concavity of

1.
2. so that are possible inflection points
3. Setting up the table:

|  |  |  |
| --- | --- | --- |
|  |  -1 ½  |  1  |
|  | - | + |
|  |  |  |

To determine the signs of the 2nd derivative, I used ‘test numbers’ in the various intervals, substituted them into the 2nd derivative and figured out the sign only.



Now is indeed an inflection point, since the concavity to its left is different from the one to its right.

**Example**: Discuss concavity for

1.
2. so that , , and are possible inflection points
3. Setting up the table:



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  -6 **-** |  -1 **0** | 1  | 6 |
|  | - | + | - | + |
|  |  |  |  |  |

To determine the signs of the 2nd derivative, I again used ‘test numbers’ in the various intervals.

Now and are *all* inflection points.