Math 1401 – Limits Worksheet

Recall that $\lim_{x\to a} f(x) = L$ means that as *x* gets closer and closer to *a* but without being equal to *a*, *f(x)* gets closer to *L.*
When you need to find a limit, you *cheat*: plug in $x = a$ (even though you're not supposed to) and hope for the best:

(i) If you get $\frac{\#}{\#}$ that would be the answer

(ii) If you get $\frac{\mathbf{0}}{\#}$ the answer is 0

(iii) If you get $\frac{\#}{\mathbf{0}}$ the answer is undefined

(iv) If you get $\frac{\mathbf{0}}{\mathbf{0}}$ tough luck, n*o info*, need *more work*

Examples: First determine if each limit is easy or tricky. Then find each limit.

a) $\lim_{x\to 0} \frac{4x-6}{2} = -3$

a') $\lim_{x\to 2} \frac{2x^2-8}{x+2} = 0$

b) $\lim_{x\to -2} \frac{2x^2-8}{x+2} = -8$

b') $\lim_{x\to -2} \frac{2x^2-4}{x+2} = undefined$

c) $\lim_{x\to -2} \frac{2x^2-8}{x-2} = 0$

c') $\lim_{x\to 2} \frac{2x^2-8}{x+2} = 0$

d) $\lim_{x\to 0} \frac{x^2-5x+6}{x^2+x-6} = -1$

d') $\lim_{x\to 2} \frac{x^2-5x+6}{x^2+x-6} = -\frac{1}{5}$

e) $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x^2} = \frac{1}{4}$

e') $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x} = 0$

f) $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x^2} = undefined$

f') $\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4} = \frac{4}{5}$

g) $\lim_{x\to -1} \frac{x^2-4x}{x^2-3x-4} = undefined$

g') $\lim_{x\to 1} \frac{x^2-4x}{x^2-3x-4} = \frac{1}{2}$

h) $\lim_{x\to 4} \frac{\frac{1}{4}-\frac{1}{x}}{4-x} = \lim_{x\to 4} \frac{\frac{x-4}{4x}}{4-x} = \lim_{x\to 4} \frac{-(4-x)}{4x(4-x)} = -\frac{1}{16}$

h') $\lim_{x\to 0} \frac{1}{x} - \frac{1}{x^2+x} = 1$

i) $\lim_{x\to \pi/2} \frac{\sin(2x)}{\sin(x)} = 0$

i') $\lim_{x\to 0} \frac{\sin(2x)}{\sin(x)} = 2$

j) If $f(x) = 2x^2 + 3$, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(2(x+h)^2+3)-(2x^2+3)}{h} = \lim_{h\to 0} \frac{2x^2+4xh+2h^2+3-2x^2-3}{h} =$

$\lim_{h\to 0} \frac{h(4x+2h)}{h} = 4x$

Recall: $\lim_{x\to a^+} f(x)$ means that *x* gets closer and closer to *a*, but *x* is always *bigger* than *a* (*x* > *a*) (right-handed).

$\lim_{x\to a^-} f(x)$ means that *x* gets closer and closer to *a*, but *x* is always *smaller* than *a* (*x* < *a*) (left-handed)

Examples: Find the domains and the limits, if they exist, for $f(x) = \begin{cases} 2x^2 + 1 & if\ x < 0 \\ 3 - x^2 & if\ x > 0 \end{cases}$ and $g(x) = \begin{cases} \frac{x^2-9}{x-3} & if\ x \neq 3 \\ -2 & if\ x = 3 \end{cases}$

a) $\lim_{x\to 1^-} f(x) = 2$

a') $\lim_{x\to 0^-} f(x) = 1$

b) $\lim_{x\to 0^+} f(x) = 3$

b') $\lim_{x\to 0} f(x)$ = undefined (different left and right limit)

c) $\lim_{x\to 3^-} g(x) = 6$

c') $\lim_{x\to 3^+} g(x) = 6$

d) $\lim_{x \to 0} g(x) = 3$

d') $\lim_{x \to 3} g(x) = 6$