

Math 1401 – Limits Worksheet

Recall that $\lim_{x \rightarrow a} f(x) = L$ means that as x gets closer and closer to a but without being equal to a , $f(x)$ gets closer to L . When you need to find a limit, you *cheat*: plug in $x = a$ (even though you're not supposed to) and hope for the best:

(i) If you get $\frac{\#}{\#}$ that would be the answer

(ii) If you get $\frac{0}{\#}$ the answer is 0

(iii) If you get $\frac{\#}{0}$ the answer is undefined

(iv) If you get $\frac{0}{0}$ tough luck, *no info*, need *more work*

Examples: First determine if each limit is easy or tricky. Then find each limit.

a) $\lim_{x \rightarrow 0} \frac{4x-6}{2}$

a') $\lim_{x \rightarrow 2} \frac{2x^2-8}{x+2}$

b) $\lim_{x \rightarrow -2} \frac{2x^2-8}{x+2}$

b') $\lim_{x \rightarrow -2} \frac{2x^2-4}{x+2}$

c) $\lim_{x \rightarrow -2} \frac{2x^2-8}{x-2}$

c') $\lim_{x \rightarrow 2} \frac{2x^2-8}{x+2}$

d) $\lim_{x \rightarrow 0} \frac{x^2-5x+6}{x^2+x-6}$

d') $\lim_{x \rightarrow 2} \frac{x^2-5x+6}{x^2+x-6}$

e) $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+4}-2}{x^2}$

e') $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+4}-2}{x}$

f) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x^2}$

f') $\lim_{x \rightarrow 4} \frac{x^2-4x}{x^2-3x-4}$

g) $\lim_{x \rightarrow -1} \frac{x^2-4x}{x^2-3x-4}$

g') $\lim_{x \rightarrow 1} \frac{x^2-4x}{x^2-3x-4}$

h) $\lim_{x \rightarrow 4} \frac{\frac{1}{4-x}}{4-x}$

h') $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{x^2+x}$

i) $\lim_{x \rightarrow \pi/2} \frac{\sin(2x)}{\sin(x)}$

i') $\lim_{x \rightarrow 0} \frac{\sin(2x)}{\sin(x)}$

j) If $f(x) = 2x^2 + 3$, find $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$

Recall: $\lim_{x \rightarrow a^+} f(x)$ means that x gets closer and closer to a , but x is always *bigger* than a ($x > a$) (right-handed).

$\lim_{x \rightarrow a^-} f(x)$ means that x gets closer and closer to a , but x is always *smaller* than a ($x < a$) (left-handed)

Examples: Find the domains and the limits, if they exist, for $f(x) = \begin{cases} 2x^2 + 1 & \text{if } x < 0 \\ 3 - x^2 & \text{if } x > 0 \end{cases}$ and $g(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ -2 & \text{if } x = 3 \end{cases}$

a) $\lim_{x \rightarrow 1^-} f(x)$

a') $\lim_{x \rightarrow 0^-} f(x)$

b) $\lim_{x \rightarrow 0^+} f(x)$

b') $\lim_{x \rightarrow 0} f(x)$

c) $\lim_{x \rightarrow 3^-} g(x)$

c') $\lim_{x \rightarrow 3^+} g(x)$

d) $\lim_{x \rightarrow 0} g(x)$

d') $\lim_{x \rightarrow 3} g(x)$