

Math 1401: Practice Exam 2

Disclaimer: This is a *practice exam* only. It is longer than the actual exam.

What is the definition of derivative $f'(x)$?

What is the Chain Rule?

What is a necessary condition for a function f to have a local extrema at $x=c$?

What can you say about a continuous function on a closed, bounded interval and absolute extrema?

What information about $f(x)$ does $f'(x)$ provide?

What information about $f(x)$ does $f''(x)$ provide?

True/False?

If $f'(x) < 0$ then f is concave down

If $f''(x) = 0$ then f has an inflection point at x .

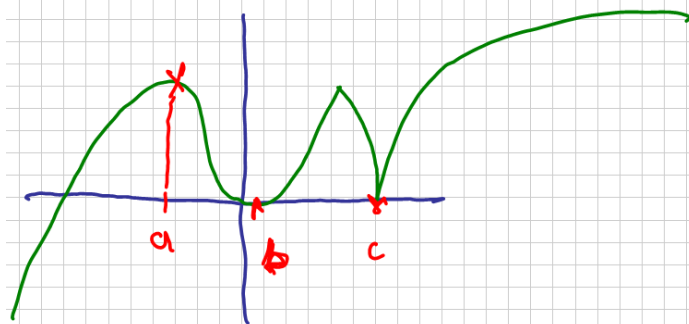
If $f'(x) = 0$ then f could have a maximum, or a minimum, or neither

If f is differentiable then f must be continuous

If $f'(x) = 0$ or $f'(x)$ is undefined, then x is called critical point

If $x=c$ is a critical point for f , then f must have a relative extrema at $x=c$

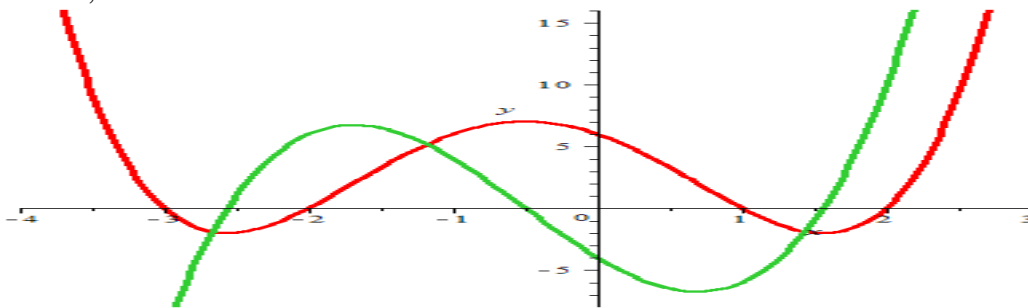
Picture Problems: Consider the graph of a function as shown:



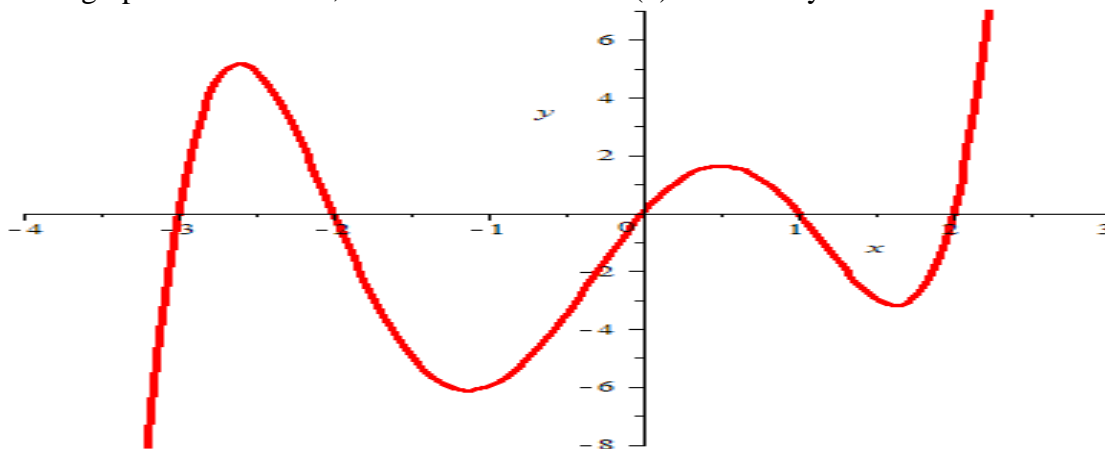
Find (sqn only) if possible

$f'(a)$	$f''(a)$
$f'(b)$	$f''(b)$
$f'(c)$	$f''(c)$

In the coordinate system below you can see the graphs of $f(x)$ and its derivative $f'(x)$. Who is the function, who is the derivative?



For the graph shown below, draw the derivative $f'(x)$ as best as you can.



Please find the derivative for each of the following functions (do not simplify unless it is helpful).

$$f(x) = x^2(x^4 - 2x)^3$$

$$f(x) = x \sin(x^2)$$

$$f(x) = \frac{\sin(x^3)}{x^4 - 3}$$

$$f(x) = \tan(x)^3 \sqrt{1-x^2}$$

$$f(x) = \pi^2 \sin\left(\sqrt{\frac{\pi}{6}}\right)$$

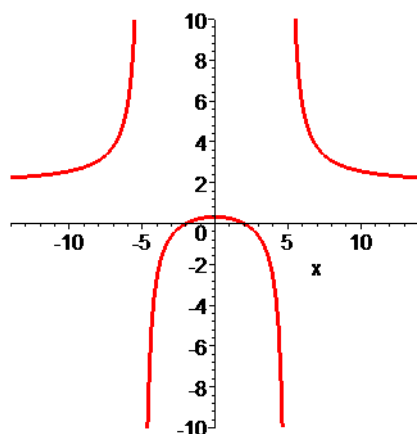
$$f(x) = \frac{x^2 \cos(1-x)}{(1-2x)^2}$$

$$f(x) = x \sin(\sqrt{1-x^2})$$

$$f(x) = \sin^2(x) + \cos^2(x)$$

$$f(x) = \tan(x^2), \text{ find } f''(x)$$

$$f(x) = \cos(x^2), \text{ find } f'''(x)$$



For the function displayed below, find the following limits:

a) $\lim_{x \rightarrow \infty} f(x)$

b) $\lim_{x \rightarrow -\infty} f(x)$

c) $\lim_{x \rightarrow 5^+} f(x)$

d) $\lim_{x \rightarrow -5^+} f(x)$

Suppose a function y is implicitly defined as a function of x via the equation $y^3 - 5x^2 = 3x$.

- Find the derivative of y using implicit differentiation.
- What is the equation of the tangent line at the point $(1, 2)$.

Find the slope of the tangent line to the graph of $y^4 + 3y - 4x^3 = 5x + 1$ at the point $(1, -2)$, assuming that the equation defines y as a function of x implicitly.

Find $\frac{dy}{dx}$ if $y = x^2 \sin(y)$, assuming that y is an implicitly defined function of x .

Find the following limits at infinity:

$$\lim_{x \rightarrow \infty} \frac{2x + 3x^4}{4x^3 - 2x^2 + x - 1}$$

$$\lim_{x \rightarrow -\infty} \frac{x - x^5}{x^3 - x^2 + x - 1}$$

$$\lim_{x \rightarrow -\infty} \frac{4x^3 - 2x^2 + x - 1}{2x - 3x^4}$$

$$\lim_{x \rightarrow -\infty} \frac{x^3 - x^2 + x - 1}{x - 3x^3}$$

$$\lim_{x \rightarrow -\infty} \frac{(3x + 4)(x - 1)}{(2x + 7)(x + 2)}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 1}}{x}$$

Find all asymptotes, horizontal and vertical, if any, for the functions

$$f(x) = \frac{3x^2 + 1}{9 - x^2}$$

$$f(x) = \frac{x^4}{1 + x^4}$$

$$f(x) = \frac{x - 3}{x^2 - 5x + 6}$$

If $f(x) = x^3 + x^2 - 5x - 5$, find the intervals on which f is increasing and decreasing, and find all relative extrema, if any.

Determine where the function $f(x) = x^4 - 2x^2$ is increasing and decreasing and find all relative extrema, if any.

Find the local maxima and minima for the function $f(x) = x^{\frac{1}{3}}(8 - x)$

Find the absolute extrema (i.e. absolute maximum and absolute minimum) for the function

$$f(x) = 3x^4 - 6x^2 \text{ on the interval } [0, 2]$$

Find the absolute maximum and minimum of the function $f(x) = 2x^3 + 3x^2 - 36x$ on the interval $[0, 4]$.

Do the same for $f(x) = \frac{x}{x^2 + 1}$ on $[0, 3]$, or for $f(x) = 3x^4 + 4x^3$ on $[-2, 0]$.

If $f(x) = x^3 + x^2 - 5x - 5$, determine intervals on which the graph of f is concave up and intervals on which the graph is concave down.

If $f(x) = 12 + 2x^2 - x^4$, find all points of inflection and discuss the concavity of f . Do the same for $f(x) = x^5 - 5x^3$,

Find the interval where $f(x) = 1 - x^{\frac{1}{3}}$ is concave up, if any.

Graph the function $f(x) = \frac{x^2}{x^2 - 1}$. Note that $f'(x) = \frac{-2x}{(x^2 - 1)^2}$ and $f''(x) = \frac{2(3x^2 + 1)}{(x^2 - 1)^3}$. Make sure to find all asymptotes (horizontal and vertical) and clearly label any maximum, minimum, and inflection points. Then do the same for the function $f(x) = \frac{8 - 4x}{(x - 1)^2}$, or $f(x) = \frac{2x^2 - 8}{x^2 - 16}$, or $f(x) = \frac{x^2 - 1}{x^3}$

A liquid form of penicillin manufactured by a pharmaceutical firm is sold in bulk at a price of \$200 per unit. If the total production cost (in dollars) for x units is $C(x) = 500,000 + 80x + 0.003x^2$ and if the production capacity of the firm is at most 30,000 units in a specified time, how many units of penicillin must be manufactured and sold in that time to maximize the profit?

A farmer wants to fence in a piece of land that borders on one side on a river. She has 200m of fence available and wants to get a rectangular piece of fenced-in land. One side of the property needs no fence because of the river. Find the dimensions of the rectangle that yields maximum area. (Make sure you indicate the appropriate domain for the function you want to maximize). Please state your answer in a complete sentence.

Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius 16, if one vertex lies on the diameter.

An open box with a rectangular base is to be constructed from a rectangular piece of cardboard 16 inches wide and 21 inches long by cutting out a square from each corner and then bending up the sides. Find the size of the corner square which will produce a box having the largest possible volume.

