Panel 1

Last Time

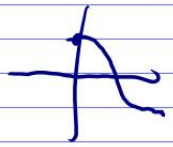
$f(x) = \sin(x), \quad x \in (-\frac{\pi}{2}, \frac{\pi}{2}) \Rightarrow \sin^{-1}(x) = y$

$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$

e.g. $\sin^{-1}(0) = y \quad |\sin(\ )$

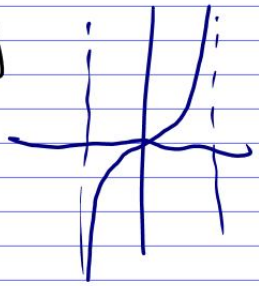
$0 = \sin(y) \Rightarrow y = 0$

$f(x) = \cos(x), \quad x \in (0, \pi)$



$\frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$

$f(x) = \tan(x) \qquad x \in (-\frac{\pi}{2}, \frac{\pi}{2})$



$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$

Panel 2

l'Hospital's Rule:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$ if $f(a) = g(a) = 0$

if $f(a) = \infty, \ g(a) = \infty$

① If $f(a) = 0, g(a) \neq 0 \Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = 0$

② If $f(a) \neq 0, g(a) = 0 \Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} =$ n.an $\begin{cases} +\infty \\ -\infty \\ \text{d.ne.} \end{cases}$

Ex: $\lim_{x \to 0} \frac{x}{\cos(x)} = 0$

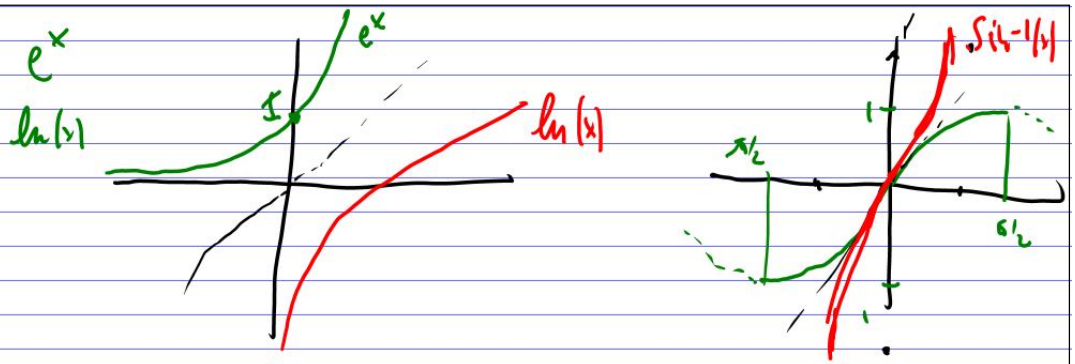
$\lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)} = \lim_{x \to 0} \frac{3 \cdot \cos(3x)}{5 \cos(5x)} = \frac{3}{5}$

$\lim_{x \to 0} \frac{3x}{\cos(4x) - 1} = \lim_{x \to 0} \frac{3}{-\sin(4x) \cdot 4} = \frac{3}{0} =$ undef

Panel 3

	Domain	Range	Derivative
e^x	$(-\infty, \infty)$	$(0, \infty)$	e^x
$\ln(x)$	$(0, \infty)$	$(-\infty, \infty)$	$1/x$
a^x	$(-\infty, \infty)$	$(0, \infty)$	$\ln(a)\, a^x$
$\log_a(x)$	$(0, \infty)$	$(-\infty, \infty)$	$1/(\ln(a)\, x)$
$\sin(x)$	$(-\pi/2, \pi/2)$	$(-1, 1)$	$\cos(x)$
$\sin^{-1}(x)$	$(-1, 1)$	$(-\pi/2, \pi/2)$	$1/\sqrt{(1-x^2)}$
$\cos(x)$	$(0, \pi)$	$(-1, 1)$	$-\sin(x)$
$\cos^{-1}(x)$	$(-1, 1)$	$(0, \pi)$	$-1/\sqrt{1-x^2}$
$\tan(x)$	$(-\pi/2, \pi/2)$	$(-\infty, \infty)$	$\sec^2(x)$
$\tan^{-1}(x)$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$	$1/(1+x^2)$

Panel 4



Panel 5

Working with our new Functions

cos	sin	θ
1	0	0
$\sqrt{3}/2$	$1/2$	$\pi/6$
$\sqrt{2}/2$	$\sqrt{2}/2$	$\pi/4$
$1/2$	$\sqrt{3}/2$	$\pi/3$
0	$\sqrt{4}/2 = 1$	$\pi/2$

Find $\frac{\pi}{6} = \cos^{-1}\left(\sqrt{3}/2\right) = y \Leftrightarrow \frac{\sqrt{3}}{2} = \cos(y)$

Solve $8e^{4x} = 3 \Rightarrow e^{4x} = \frac{3}{8}$ | ln

$\ln(e^{4x}) = 4x = \ln(3/8)$

$x = \frac{1}{4}\ln(3/8)$

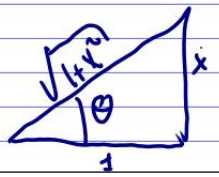
Solve $6\sin(2x) = 3$

$\sin(2x) = \frac{1}{2}$ | $\sin^{-1}$

$2x = \sin^{-1}(1/2)$

$2x = \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\cdot\frac{\pi}{6} = \frac{\pi}{12}$

Simplify $\sin\left(\tan^{-1}(x)\right) = \frac{\text{opp}}{\text{hyp}} = \frac{x}{\sqrt{1+x^2}}$



$\theta = \tan^{-1}(x)$

$\tan(\theta) = x = \frac{x}{1}$

Panel 6

Find the following derivatives

$f(x) = x^2\sin^{-1}(x)$

$f'(x) = 2x\sin^{-1}(x) + x^2\frac{1}{\sqrt{1-x^2}}$

$f(x) = e^x \cdot \ln\left(\cos^{-1}(x)\right)$

$f'(x) = e^x\cdot\ln\left(\cos^{-1}(x)\right) + e^x\frac{1}{\cos^{-1}(x)}\cdot\frac{(-1)}{\sqrt{1-x^2}}$

$0 + \pi/2 = c$

$\sin^{-1}(0) + \cos^{-1}(0) = c$

$f(x) = \sin^{-1}(x) + \cos^{-1}(x) \Rightarrow \sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$

$f'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}} \equiv 0$

$\sin^2(x) + \cos^2(x) = 1$

$y = f(x) = \frac{x^3\sqrt{1-x}}{\sin^2(x)\cdot e^x}$

$\ln(y) = 3\ln(x) + \frac{1}{2}\ln(1-x) - 2\ln(\sin(x)) - \ln(e^x)$

$\frac{1}{y}\cdot y' = \frac{3}{x} - \frac{1}{2(1-x)} - 2\cot(x) - 1$

Panel 7

Exponential Growth / Decay

Carbon Dating: ^{14}C has half-life of 5730 years. How much ^{14}C is left after 1500 years if you started out with 100g. First estimate (12), then compute exact value.

$A = A_0 e^{kt}$, A_0 = initial amount, i.e. $A_0 = 100$

$100 e^{k \cdot 5730} = A(5730) = 50 = 100 e^{k \cdot 5730} \Rightarrow \frac{1}{2} = e^{k \cdot 5730}$

$\Rightarrow \ln(\frac{1}{2}) = k \cdot 5730 \Rightarrow k = \frac{1}{5730} \cdot \ln(\frac{1}{2}) = -0.0001$

$A(t) = 100 e^{-0.0001t}$

$A(5730) = 100 e^{-5730 \cdot 0.0001} = 50.2$

$A(1500) = 100 e^{-1500 \cdot 0.0001} = 86.1$

Panel 8

How old is paper containing 74% of usual amount of ^{14}C?

$A(t) = A_0 e^{-0.0001t}$

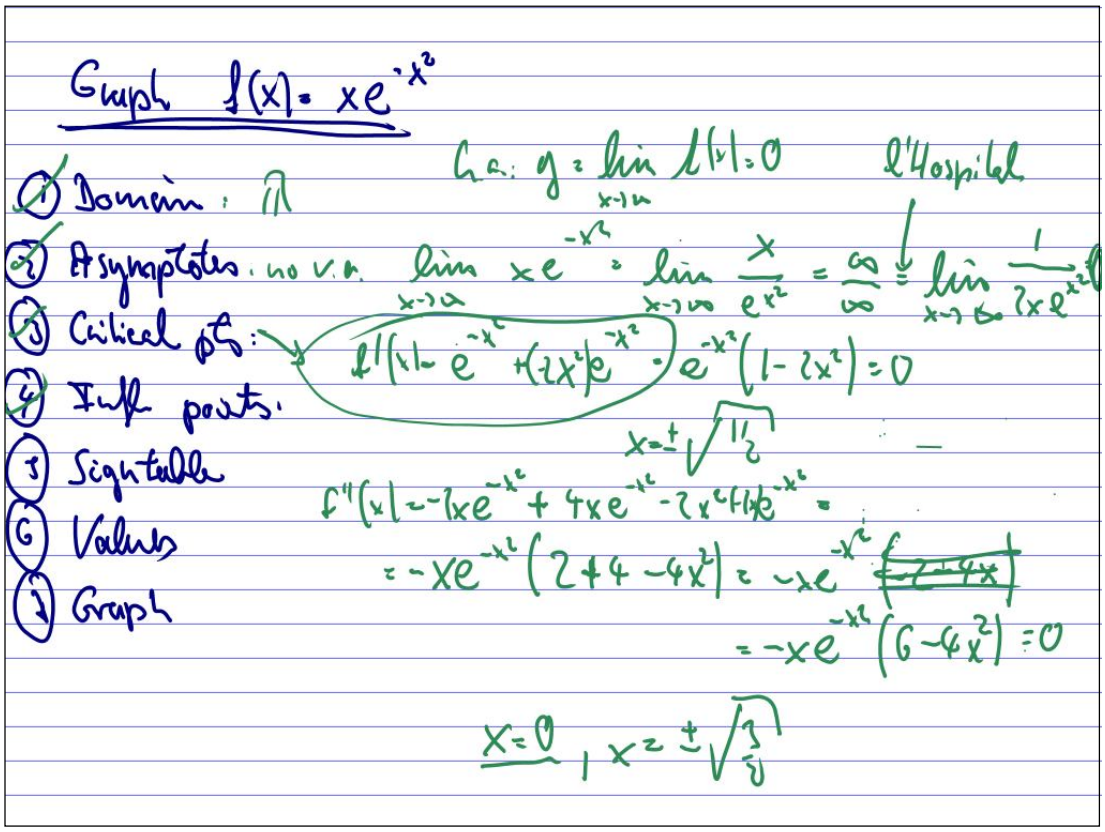
$A(t) = 0.74 \cdot A_0 = A_0 e^{-0.0001t}$

$0.74 = e^{-0.0001t}$ | ln( )

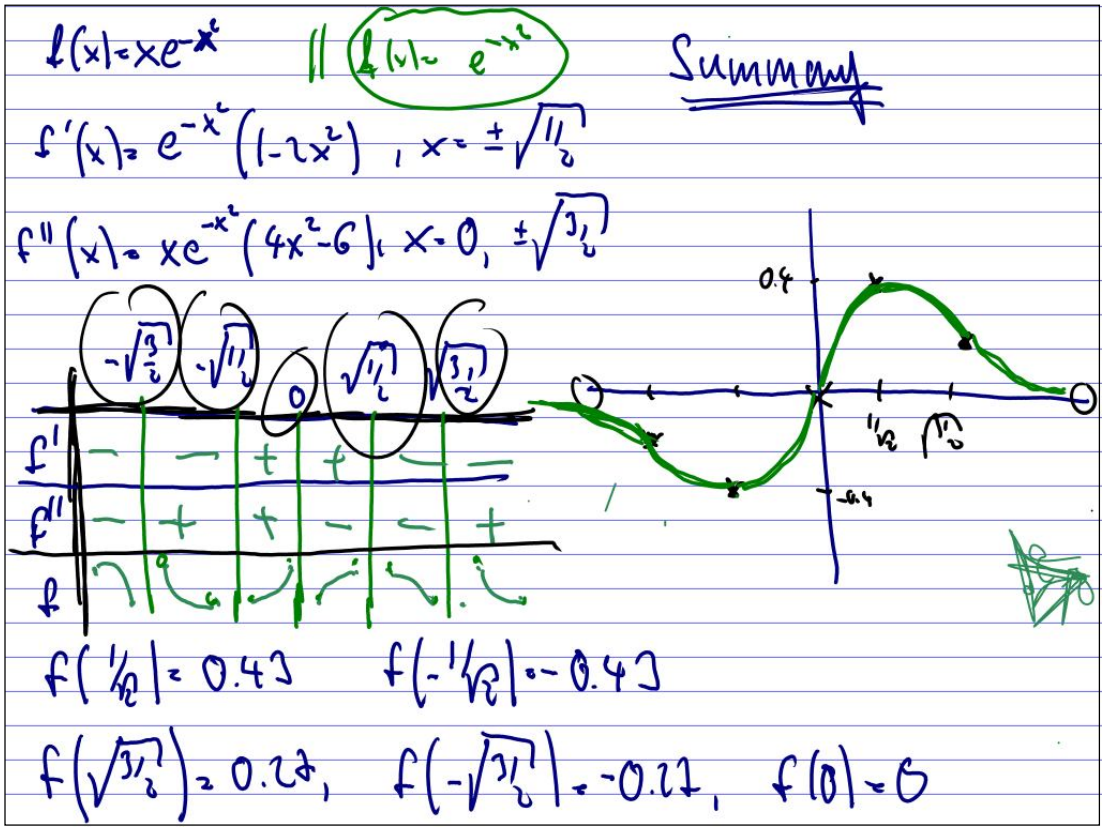
$\ln(0.74) = -0.0001t$

$\frac{\ln(0.74)}{0.0001} = t = 2709$ years old

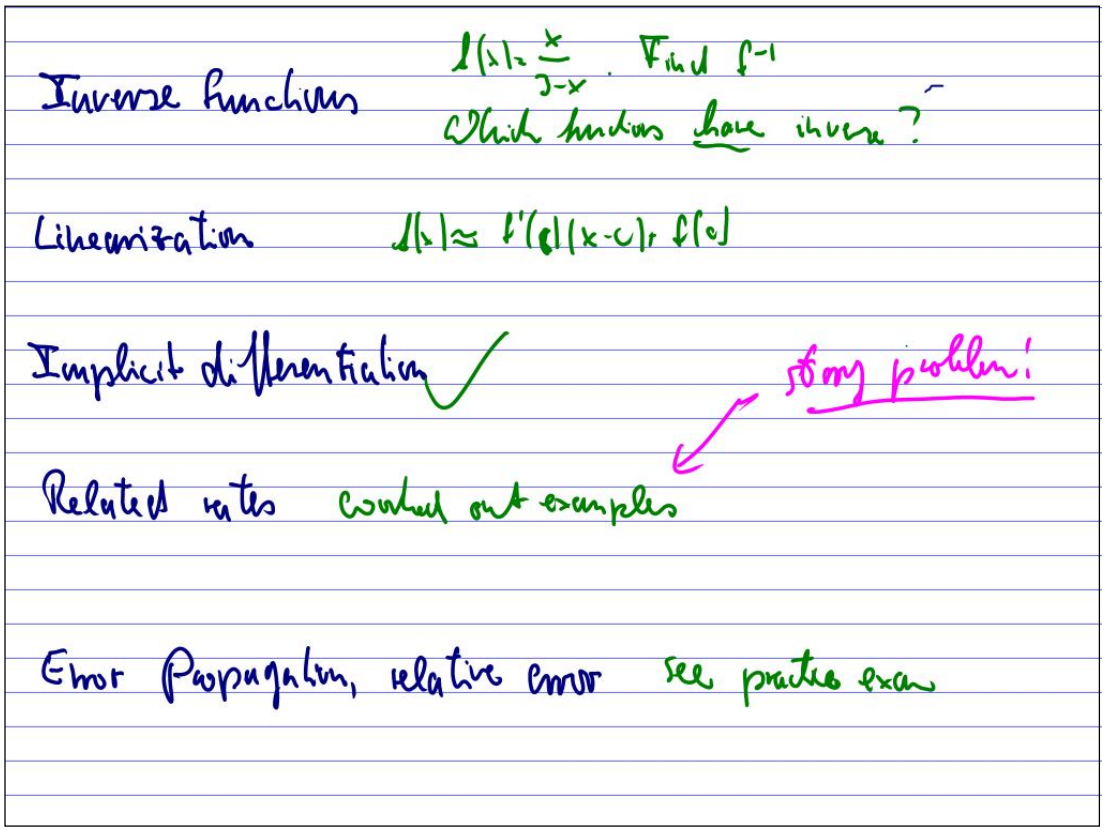
Panel 9



Panel 10



Panel 11


Inverse functions
$f(x)=\frac{x}{3-x}$. Find f^{-1}
Which functions have inverse?
Linearization
$f(x)\approx f'(a)(x-c)+f(a)$
Implicit differentiation
Some problem!
Related rates
Counter examples
Error Propagation, relative error
see practice exam