Panel 1

Summary of Applications of Derivatives

1. Curve sketching
2. Optimization (max/min)
3. Implicit Diff
4. Related Rates
5. Linearization
6. Error estimates

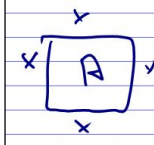
Panel 2

Related Rates: Find an equation relating 2 variables, both are (usually) functions of t.
$\Rightarrow \frac{d}{dt}$ to relate the rates (derivatives)

Question:

3. Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2?

4. The length of a rectangle is increasing at a rate of 8 cm/s



$\frac{d}{dt}(A = x^2)$

$A' = 2x \cdot x'$

Know: $x' = 6$

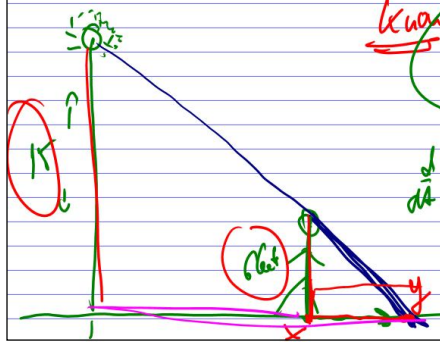
Want: $A' = ?$

$= 2 \cdot x \cdot x'$

when $A = 16 \Rightarrow x = 4$ $= 2 \cdot 4 \cdot 6$

Panel 3

A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?



Know $\frac{15}{6} = \frac{x+y}{x}$

$15x = 6x + 6y$

$\frac{d}{dt}$

$9x = 6y$

$9x' = 6y' \Rightarrow \frac{9x'}{6} = y'$

$\frac{3}{2} \cdot 5 = y'$

Panel 4

. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

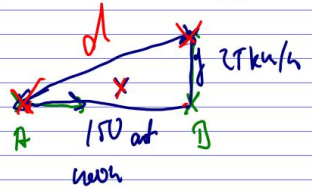
Want d'

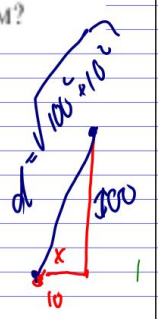
$\frac{d}{dt}(d^2 = x^2 + y^2)$

$2d \cdot d' = 2x \cdot x' + 2y \cdot y'$

Know $x' = 35$

$y' = 25$





4 hours A traveled $4 \cdot 35 = 140$ miles

B traveled $4 \cdot 25 = 100$ miles

Panel 5

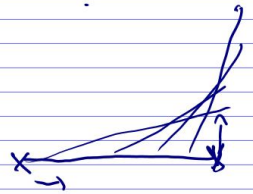
$dd' = xx' + yy'$

$d = \sqrt{100^2 + 10^2} = 100.5$

$x = 10$ $\rightarrow$ $100.5 \cdot d' = 10 \cdot (-35) + 100 (2\pi)$

$y = 100$

$d' = \frac{-100}{100.5} = -0.99$ km/h



Panel 6

Linearization

Replace complicated function by easy one, say a linear one.

$f(x) - f(c) \approx f'(c)(x-c)$

$\left(\lim \frac{f(x) - f(c)}{x - c} \approx f'(c)\right)$

$f(x) \approx f'(c)(x-c) + f(c)$

Ex: Linearize $f(x) = 2x + \cos(x)$ near 0

$f'(x) = 2 + \sin(x)$ $\rightarrow f'(0) = 2$, $f(c) = 1$

Panel 7

$\rightarrow 2x + \cos(x) \approx 2(x-0) + 1 = 2x + 1$

Calc: $2 \cdot 0.1 + \cos(0.1)$

$1.195 = 0.2 + \cos(0.1) \approx 2 \cdot 0.1 + 1 = 1.2$

$c = 0$

$f(x) = \sqrt{(2 + x^2)\cos(x)}$ $f(0.1) = ?$

$f'(x) = \frac{1}{2}\left((2+x^2)\cos(x)\right)^{-1/2} \cdot (2x\cos(x) + (2+x^2) \cdot (-\sin(x)))$

$f'(0) = \frac{1}{2}(2)^{-1/2} \cdot 0 = 0$ $f(x) \approx f'(0)(x-0) + f(0)$

$f(0) = \sqrt{2}$ $1.4146 \; f(0.1) \approx 0 + \sqrt{2} \sim 1.41$

Panel 8

Linearization: take c, find $f'(c)$ and $f(c)$

$\rightarrow f(x) \approx f'(c) \cdot (x-c) + f(c)$

Find $\sin(3)$: $\sin(x)$ $c = \pi$

$\sin(x) \approx \cos(c)(x-c) + \sin(c)$

$\sin(x) \approx -1(x-\pi) + 0$

$\rightarrow \sin(3) \approx -1(3-\pi) = +0.14$

0.1411

Panel 9

Error Estimation

Start with $f(x) = y$

$\frac{dy}{dx} = f'(x) \Rightarrow dy = f'(x) \cdot dx$

Measure diam. of Silver $: 1.7$ inches

$\rightarrow r = 0.7 \pm 0.1$

$\rightarrow A = \pi r^2 = 1.766$

dr impact on dA

$A = \pi r^2$

$dA = \pi 2r(dr)$

$= \pi 2 \cdot 0.7 \cdot 0.1 = 0.471$

Panel 10

Relative Error: $\frac{dr}{r}$

$r = 1.7 \pm 0.1, \quad dr = 0.1 \Rightarrow \frac{dr}{r} = \frac{0.1}{1.7} = 0.05$ or 5%

$A = \pi r^2, \quad dA = 2\pi r\, dr, \quad \frac{dA}{A} = \frac{0.471}{1.766} = 0.26$

or 26%

Panel 11

Quiz #7 Name:

(1) Find the linearization of $f(x) = \sqrt{2+x}$ near $c = 2$ and use it to estimate $\sqrt{4.1}$

(2) The radius of a disk is measured to be 24 cm
Estimate the error in computing area of the di

+ relative error

Panel 12

3 Two cars start from the same spot. One travels
other goes north at 25 mph. At what rate is the
the two cars increasing two hours later ?

Quiz on Monday