Panel 1

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

If $f'(x) = 0$ then f may or may not have min/max

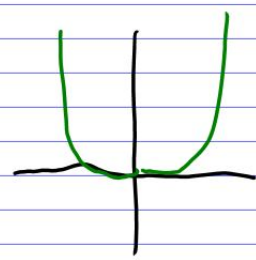
If f has a min/max, then $f'(x) = 0$

$f(x) = x^6$

$f'(x) = 6x^5$

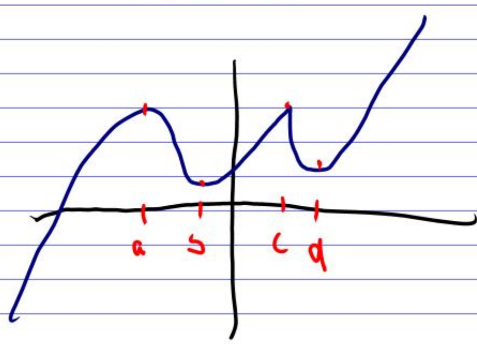
$f''(x) = 30x^4$

$f''(0) = 0$



Panel 2

Picture Problems



$f'(a) = 0$ $f''(a) < 0$

$f'(b) = 0$ $f''(b) > 0$

$f'(c)$ d.n.e. $f''(c)$ d.n.e

$f'(d) = 0$ $f''(d) > 0$

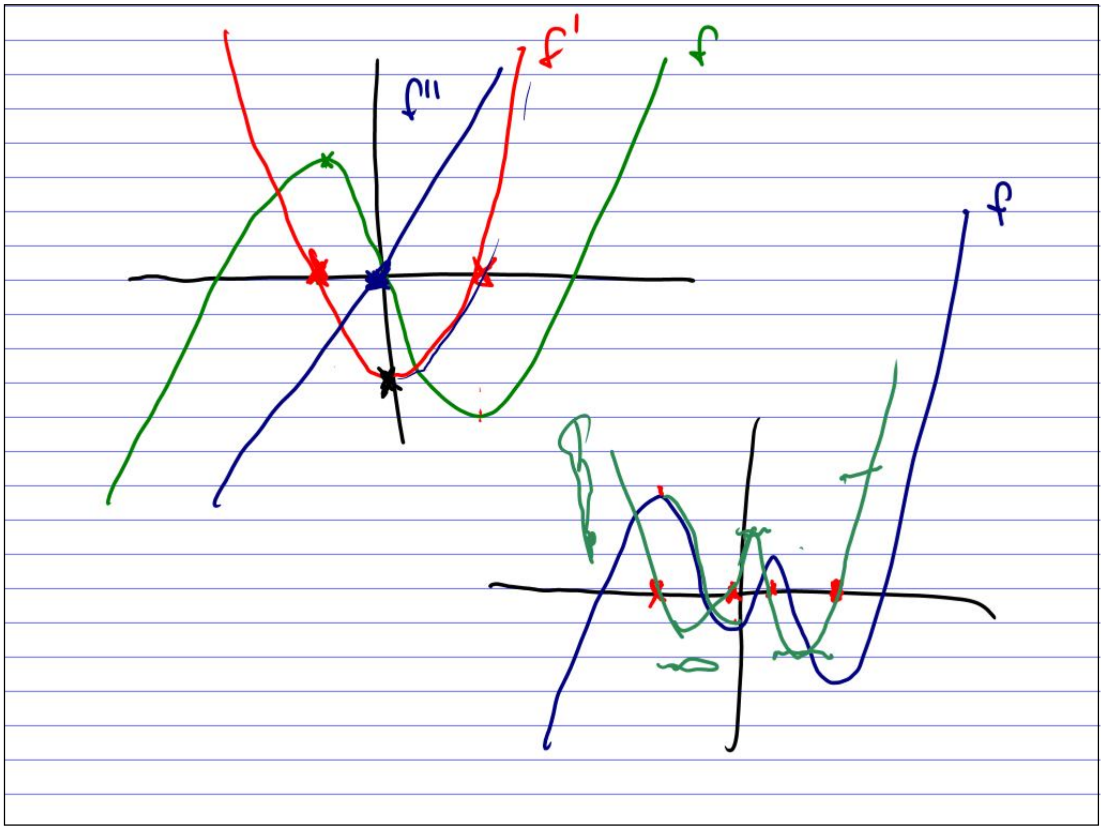
(Signs only)

ideas: ⇒ Graph with 2 or 3 functions:

- which one is f, or f', or f''

→ Graph of $f(x)$. Sketch f' (or f'')

Panel 3



Panel 4

$f(x) = \cos^2(x) + \sin^2(x) = 1$ $\qquad f'(x) = 0$

$f'(x) = 2\cdot(\cos(x))(-\sin(x)) + 2\,[\sin(x)]\cdot\cos(x) = 0$

$f'(x) = 0$ for all x proves that $\cos^2(x) + \sin^2(x) =$ const.

(const $= 1$)

To find horizontal asympt.: $y = \lim_{x \to \pm\infty} f(x)$

vertical asympt.: $\lim_{x \to c^{+/-}} f(x) = \pm\infty$

Ex: $f(x) = \frac{x^2-4}{x^2-3x+2}$ $\qquad \lim_{x \to \pm\infty} \frac{x^2-4}{x^2-3x+2} = 1 = y$ is ha

$(x-2)(x-1)$

v.a. $x = 2$, $x = 1$ because $\lim_{x \to 1} f(x) = \pm\infty$

Same!

Panel 5

$f(x) = x^3 + x^2 - 5x - 5$ decr/incr. ?

$f'(x) = 3x^2 + 2x - 5 = 0$

$x_{1/2} = \dfrac{-2 \pm \sqrt{4+60}}{6} = \dfrac{-2 \pm 8}{6} = \begin{cases} \frac{-10}{6} = -5/3 \\ \frac{+6}{6} = +1 \end{cases}$

	$-\frac{5}{3}$	0	$+1$	
f'	+	−		+
f	↗	↘		↗

f is incr. on $(-\infty, -5/3)$ and on $(1, \infty)$.

Decreasing on $(-\frac{5}{3}, 1)$

Panel 6

$f(x) = 3x^4 - 6x^2$ abs extrema on $[0, 2]$

$f'(x) = 12x^3 - 12x = 0 = 12x(x^2-1)$ $x = 0, x = \cancel{-1}, 1$

x	$f(x)$	
0	0	max
1	−3	
2	6	min

$f''(x) = 36x^2 - 12 = 12(3x^2 - 1) = 0$

$3x^2 = 1$

$x^2 = 1/3$

$x = \pm\sqrt{1/3}$

Panel 7

Sketch $f(x) = \frac{x^2-1}{x^3}$

$f'(x) = \frac{3-x^2}{\boxed{x^4}}$ +

$f''(x) = \frac{2(x^2-6)}{x^5}$ +

+

① Domain: $x \neq 0$

② h.a.: $y = 0$

v.a.: $x = 0$

③ $f'(x) = 0$: $x = \pm\sqrt{3}$

$f''(x) = 0$: $x = \pm\sqrt{6}$

$f(-\sqrt{6}) = 5/-14.69 = -0.3$

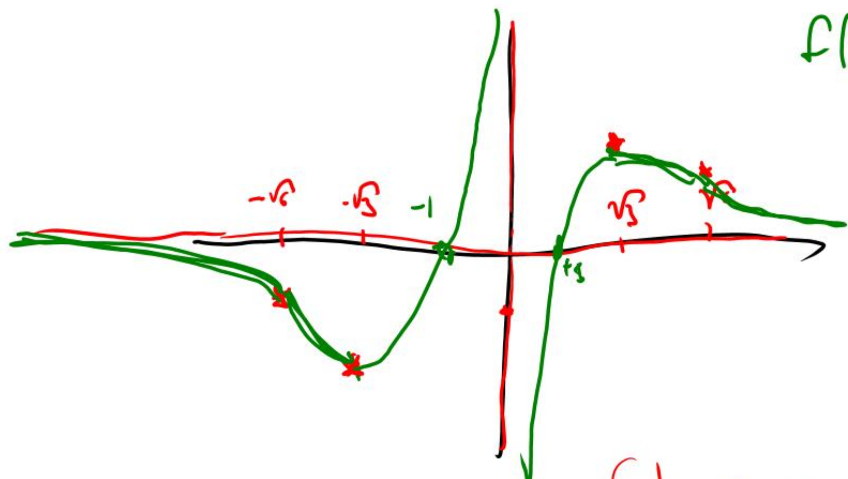
$f(-\sqrt{3}) = 2/-5.2 = -0.38$

$f(\sqrt{3}) \approx 0.38$

$f(\sqrt{6}) \approx 0.3$

	$-\sqrt{6}$	$-\sqrt{3}$	0	$\sqrt{3}$	$\sqrt{6}$
	(-3)	(-2)	(-1)	0	(1)	(2)
f'	−	−	+	+	−	−
f''	−	+	+	−	−	+

Panel 8



$f(x) = \frac{x^2-1}{x^3}$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm\sqrt{1}$

$f(-\sqrt{6}) = 5/-14.69 = -0.3$

$f(-\sqrt{3}) = 2/-5.2 = -0.38$

$f(\sqrt{3}) \approx 0.38$

$f(\sqrt{6}) \approx 0.3$

	$-\sqrt{6}$	$-\sqrt{3}$	0	$\sqrt{3}$	$\sqrt{6}$
f'	−	−	+	+	−	−
f''	−	+	+	−	−	+

Panel 9

$y^3 - 5x^2 = 3x$ defines y as function of x

a) Find y': $\quad y^3 - 5x^2 = 3x \quad \Big| \frac{d}{dx}$

$\frac{d}{dx}\left(y^3 - 5x^2\right) = \frac{d}{dx}(3x)$

$3y^2 \cdot y' - 10x = 3 \rightarrow 3y^2 y' = 3 + 10x$

$y' = \frac{3+10x}{3y^2}$

b) Equation of tangent at $(1,2)$

$y - 2 = \frac{13}{12}(x - 1)$

$y' = \frac{3+10}{3 \cdot 4} = \frac{13}{12}$

Panel 10

$y = x^2 \sin(y) \qquad$ Know: $y = y(x)$

y': $\quad \frac{d}{dx} y = \frac{d}{dx}\left(x^2\right)\sin(y)$

$y' = 2x \cdot \sin(y) + x^2 \cos(y) \cdot y'$

$y' - y' x^2 \cos(y) = 2x \sin(y)$

$y'\left(1 - x^2 \cos(y)\right) = 2x \sin(y)$

$y' = \frac{2x \sin(y)}{1 - x^2 \cos(y)}$

Panel 11

$y = x^2 \sin(y)$. Say $x = x(y)$

$\frac{d}{dy}(y) = \frac{d}{dy} x^2 \sin(y)$

$1 = 2x \cdot x' \sin(y) + x^2 \cos(y)$

$1 - x^2\cos(y) = 2x\,x' \sin(y)$

$\frac{1 - x^2\cos(y)}{2x \sin(y)} = x'$