Panel 1

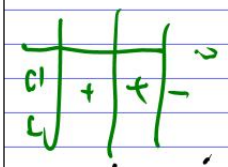
So far:

f cont on [a,b]:

ABS extrema: $\Rightarrow f'(x)=0$ $\Rightarrow$ critical points

x	f(x)
critical	⇐ largest / smallest
endpoints	

(rel. extrema)

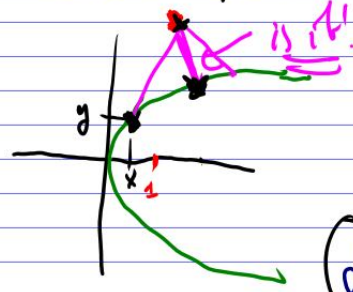


Optimization: Want to find the largest, smallest, fastest, cheapest, etc.

Panel 2

Find the point on $y^2 = 2x$ that is closest to (1,4).

$\rightarrow x = \frac{1}{2}y^2$



Want to minimize distance between (1,4) and graph $y^2 = 2x$

$d = \sqrt{(1-x)^2 + (4-y)^2}$

$d = \sqrt{(1-\frac{1}{2}y^2)^2 + (4-y)^2}$ smallest if d^2 is!

$f(y) = (1-\frac{1}{2}y^2)^2 + (4-y)^2$ to be minimized

$f = 1 - y^2 + \frac{1}{4}y^4 + 16 - 8y + y^2$

$f' = -2y + y^3 - 8 + 2y = y^3 - 8 = 0 \Rightarrow y = 2$

min $d = \sqrt{2}$ at (2,2)

$y^2 = 2x \Rightarrow 4 = 2x \Rightarrow x = 2$

Panel 3

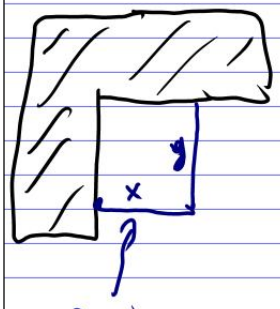
Optimization

Blah blah... long story... blah **maximize (or minimize)** blah blah...

- Draw picture ✓
- Determine variables ✓
- Label picture ✓
- Setup equations of what you know ✓
- Setup equation of what you want!
- Take critical pts.

Panel 4

Plot of land in the corner of two buildings. I have 100 m of fencing available. What is largest possible plot? Find max. area!



x = width, y = length

Know: $x + y = 100 \Rightarrow x = 100 - y$

Want: max. $A = xy = (100-y)\cdot y$

$A(y) = 100y - y^2$, $y \in [0, 100]$

$A'(y) = 100 - 2y = 0 \Rightarrow y = 50 \Rightarrow x = 50$

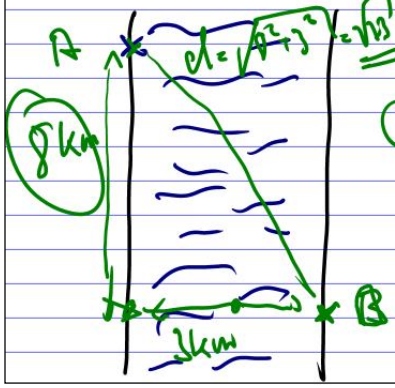
	y	A
endpoints	50	2500
	0	0
	100	0

The max area is 2500 m², for $x = 50$ m and $y = 50$ m

Panel 5

EXAMPLE 4 A man launches his boat from point A on a bank of a straight river, 3 km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible (see Figure 7). He could row his boat directly across the river to point C and then run to B, or he could row directly to B, or he could row to some point D between C and B and then run to B. If he can row 6 km/h and run 8 km/h, where should he land to reach B as soon as possible? (We assume that the speed of the water is negligible compared with the speed at which the man rows.)

Blah, blah, ..., minimize time:

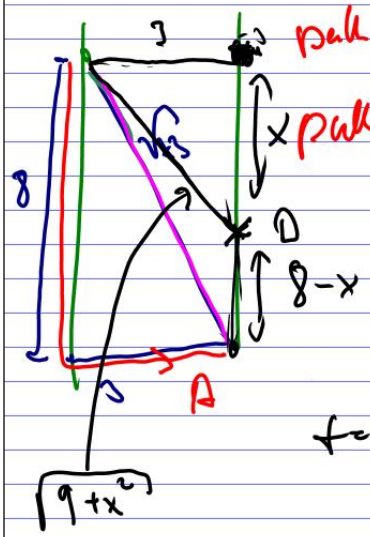


① Row all the way: Row = 6 km/h $\Rightarrow t = \sqrt{13}/6$, $t = 1.62$

② Run for 1km, then swim (by boat) for 3km.
$t_1 = 1/8 = 3/24$ $t_2 = 3/6$ $\Rightarrow t = 1 + 1/2 = 1.5$

speed = d/t $\Rightarrow t = d/\text{speed}$.

Panel 6



path A: $t = 8/8 + 3/6 = 1.5$

path D: $t = \sqrt{13}/6 \approx 1.4$

$$t = \underbrace{\frac{\sqrt{9+x^2}}{6}}_{\text{boat}} + \underbrace{\frac{8-x}{8}}_{\text{land}}, \quad x \in [0, 8]$$

$t = d/s$ $f(x) = \frac{\sqrt{9+x^2}}{6} + \frac{8-x}{8}$ minimize

$$f'(x) = \frac{1}{6}\cdot\frac{1}{2}(9+x^2)^{-1/2}\cdot 2x + \frac{1}{8}(-1) = 0$$

$$\frac{1}{6}\cdot\frac{x}{(9+x^2)^{1/2}} = \frac{6}{8} = \frac{3}{4}$$

Running speed: 8

Rowing speed: 6

Panel 7

$x = \frac{3}{4}\sqrt{9+x^2}\ /(\ )^2$

$x^2 = \frac{9}{16}(9+x^2)$

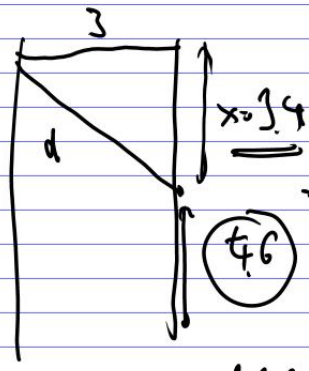
$x^2 = \frac{81}{16} + \frac{9}{16}x^2$

$x^2 - \frac{9}{16}x^2 = \frac{81}{16}$

$x^2\frac{1}{16} = \frac{81}{16}$

$x^2 = \frac{81}{1}$

$x = \sqrt{\frac{81}{1}} = 3.4$



$d = \sqrt{3^2+3.4^2}$

$= 4.93$

$\Rightarrow t = 4.93/6 = 0.821$

$t = 4.6/3 = 0.821$

total wire length is 1.93

Panel 8

Business Math

If $C(x)$ is the cost of producing x units

$\Rightarrow C'(x)$ is marginal cost

If $p(x)$ is price per unit then

$R(x) = x \cdot p(x)$ is Revenue

R' is marginal revenue

Profit $P(x) = R(x) - C(x)$

P' is marginal profit.

Panel 9

Ex 1: If $C(x) = 16{,}000 + 200x + 4x^{3/2}$, find the cost and the marginal cost at a production level of $x = 1000$

$C(1000)$ $\quad C'(1000)$: $\quad C'(x) = 200 + 4 \cdot \frac{3}{2} x^{1/2}$ $\quad C'(1000) = 200 + 6(1000)^{1/2} = 389.74$

Ex 2: If $C(x) = 16{,}000 + 500x - 1.6x^2 + 0.004x^3$ is the cost, and $p(x) = 1{,}700 - 7x$ is the demand function, (price per unit) find production level to max. profit.

$P(x) = R(x) - C(x) =$ (price is \$ $1700 - 7x$ / sell x items) $R(x) = x(1700 - 7x)$

$= x(1700 - 7x) - 16{,}000 - 500x + 1.6x^2 - 0.004x^3$

Panel 10

$1700x - 7x^2 - 16{,}000 - 500x + 1.6x^2 - 0.004x^3 = P(x)$

$P'(x) = 1700 - 14x - 500 + 3.2x - 0.012x^2$

$= 1200 - 10.8x - 0.012x^2 = 0$ $\quad a = -0.012$, $b = -10.8$, $c = 1200$

$x = \dfrac{10.8 \pm \sqrt{10.8^2 + 4 \cdot 1200 \cdot 0.012}}{-0.024}$

$\dfrac{10.8 \pm 13.6}{-0.024} = \dfrac{24}{-0.024}$ and $\dfrac{-2.8}{-0.024} = 116$

$= -1000$

Produce 116 items for max. profit.

Panel 11

Implicit Differentiation

$y = f(x)$ explicitly defines y as a function of x.

$f(x,y) = c$ implicitly defines y as a function of x or x as a function of y.

Ex: $y = 2x^2 + 7$ explicit $y' = 4x$

$x^2 + y^2 = 9$ implicit $y' = ?$

$x^2 + [y(x)]^2 = 9 \quad \left| \frac{d}{dx} \right.$

$2x + 2y \cdot y' = 0 \Rightarrow 2x + 2yy' = 0 \Rightarrow y' = \frac{-2x}{2y} = -\frac{x}{y}$

Panel 12

Ex: $x^3 + y^3 = 6xy$. Find $\frac{dy}{dx}$ if $y = y(x)$

Cannot find $y(x) = ?$ But I can find $y' = \frac{dy}{dx}$

$x^3 + y^3 = 6xy \quad \left| \frac{d}{dx} \right.$

$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = \frac{d}{dx}(6xy)$

$3x^2 + 3y^2 y' = 6y + 6x y'$

$3x^2 - 6y = 6xy' - 3y^2 y'$

$= y'(6x - 3y^2) \Rightarrow y' = \frac{3x^2 - 6y}{6x - 3y^2}$