Panel 1

So far:

local max/min, incr./decr. } $f' = 0$

f'	
f	

inflection points, concavity } $f'' = 0$

f''	
f	

Sketch:

$f' = 0$

$f'' = 0$

f'	
f''	
f	

+ asymptotes etc

Panel 2

Investigate concavity of $f(x) = x^4 - 12x^2$

$f'(x) = 4x^3 - 24x$

$\Rightarrow f''(x) = 12x^2 - 24 = 0$

$= 12(x^2 - 2) = 0$

$\Rightarrow x = \pm\sqrt{2}$

	-1000 ($-\sqrt{2}$)	0 ($\sqrt{2}$)	1000
f''	$+$	$-$	$+$
f	$\cup$	$\cap$	$\cup$

concave up   concave down

Panel 3

Sketch $f(x) = \frac{x}{x^2-4}$ $\qquad f'(x) = -\frac{x^2+2}{(x^2-4)^2} = 0$

Domain: $x \neq \pm 2$

$f'(x) = 0$, f' d.n.e.: $x = \pm 2$ $\qquad f''(x) = \frac{2x(x^2+3)}{(x^2-4)^3} = 0$

$f''(x) = 0 \Leftrightarrow x = 0$

f'' d.n.e.: $x = \pm 2$

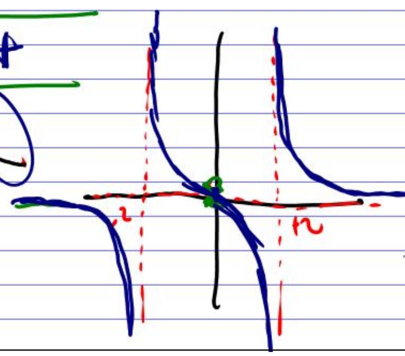
-1000 ↓, -2, -1 ↓, 0, +1 ↓, 2, 100 ↓

	-1000	-1	$+1$	100
f'	$-$	$-$	$-$	$-$
f''	$-$	$+$	$-$	$+$
f				

Asymptotes:

$x = \pm 2$ (vertical)

$\lim_{x \to \pm\infty} \frac{x}{x^2-4} = 0 = y$ (hor)



$f(0) = 0$

Panel 4

Quiz #6 Name: ______

① Find all relative extrema of $f(x) = x^3 + 3x^2 - 9x + 3$

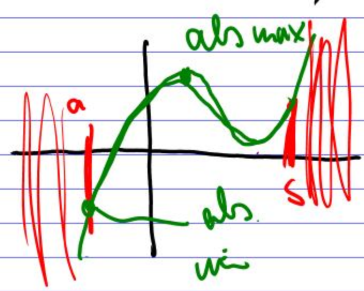
② Find all inflection points of $f(x) = 2x^3 + 6x^2 - 9$

Panel 5

(3) Sketch $f(x) = \frac{x^2}{x^2 - 1}$ Use $f'(x) = -\frac{2x}{(x^2-1)^2}$, $f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3}$

Panel 6

<u>Absolute Extrema</u>



relative or local

Abs. max: if $f(x_0) \geq f(x)$

for <u>all</u> x

near x_0

Abs min: if $f(x_0) \leq f(x)$

rel or local

for all x

near x_0

<u>Thm:</u> If $f(x)$ is continuous on a closed, bounded interval $[a,b]$ then it <u>must</u> have abs. max + min !!! Moreover, they can occur only at critical points or endpoints.

Panel 7

Ex: Find abs. extrema for $f(x) = 3x^4 - 6x^2$, $x \in [0,2]$

Know: there is abs. max, abs. min!
that they occur at critical points or endpoints

Critical: $f'(x) = 12x^3 - 12x = 0$

$12x(x^2-1) = 0 \Rightarrow x = 0, -1, 1$

between 0, 2.

x	$f(x)$
0	0
1	-3
$\cancel{-1}$	
$\cancel{2}$	
2	32

abs max. occurs at $x=2$ and is 32.
abs. min occurs at $x=1$ and is -3

Panel 8

Ex: Take $f(x) = \frac{x}{x^2+1}$ for $x \in [0,3]$. Find abs. extrema.

There is abs. max:
abs. min:

outside

$$\frac{d}{dx}\left(\frac{x}{x^2+1}\right) = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = 0 \qquad x = 1, -1$$

x	
1	$1/2$
0	0
3	$3/10$

max
min!

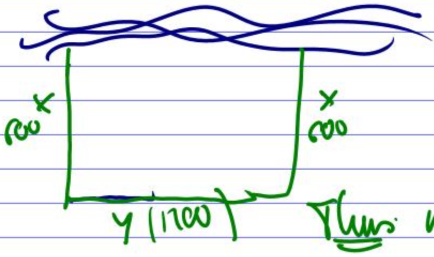
Panel 9

Optimization Problems

max →

x	A
600	720000
0	0
1200	0

Typical problem: A farmer has 2400 feet of fencing. She wants to enclose a rectangular area bordering a river. She needs no fencing along the river. Dimension of plot with max. area?



Know: $2x + y = 2400$ → $y = 2400 - 2x$

Want: max $A = xy$

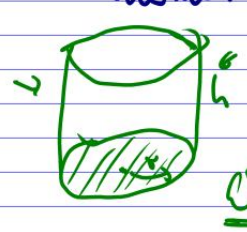
$= x(2400 - 2x)$

Thus need A'!

→ $A = 2400x - 2x^2$ ⇒ $A'(x) = 2400 - 4x = 0$ $x = 600$

Panel 10

Ex: A cylindrical can needs to hold 1 l of oil. Find the dimensions that minimize the cost of the metal.



$V = h\pi r^2 = 1$ ⇒ $h = \frac{1}{\pi r^2}$

$S = 2\pi r h + 2\pi r^2 = 2\pi r\left(\frac{1}{\pi r^2}\right) + 2\pi r^2$

Want: minimize $S = \frac{2}{r} + 2\pi r^2$

$S'(r) = 2(-1)r^{-2} + 4\pi r = 0$

$h = \frac{1}{\pi}\left(\frac{1}{2\pi}\right)^{2/3}$

$-\frac{2}{r^2} + 4\pi r = 0$

$4\pi r = \frac{2}{r^2}$

$4\pi r^3 = 2$ → $r^3 = \frac{2}{4\pi} = \frac{1}{2\pi}$

→ $r = \sqrt[3]{\frac{1}{2\pi}}$