Sample Quiz

1. Consider the function $f(x) = x^4 - 2x^2 + 1$. Determine and classify the local *extrema* and intervals where f is *increasing or decreasing*.

$f(x) = x^4 - 2x^2 + 1$.

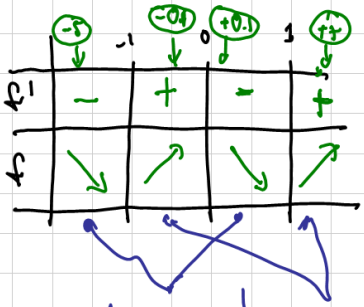
$f'(x) = 4x^3 - 4x =$

$= 4x(x^2 - 1) = 0$

$\Rightarrow x = 0,\ x = -1,\ x = +1$

are critical.

To check for local extrema, we need $f'(x)$. So:



$\Rightarrow$ $x = -1, +1$ are local minima

$x = 0$ is a local max

decreasing! on $(-\infty, -1) \cup (0, 1)$

increasing: $(-1, 0) \cup (1, \infty)$

2. Consider the function $f(x) = x^4 - 6x^2$. Find the intervals of *concavity* and identify all *inflection points*, if any.

$f(x) = x^4 - 6x^2$.

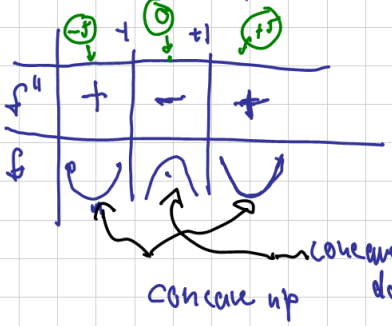
$f'(x) = 4x^3 - 12x$

$f''(x) = 12x^2 - 12$

$= 12(x^2 - 1) = 0$

$x = +1, -1$

To check for inflection points we need $f''(x)$. So:



$\Rightarrow$ So $x = +1, -1$ are indeed inflection points (because f does change direction)

3. Sketch the function $f(x) = \frac{x^2}{x^2 - 4}$, including all asymptotes, extrema, inflection points, intervals of increase, decrease, concave up, and concave down

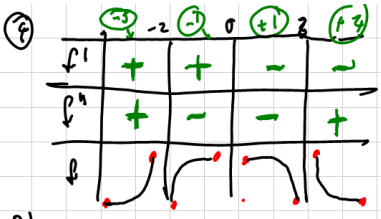
$f(x) = \frac{x^2}{x^2 - 4}$

$f'(x) = -\frac{8x}{(x^2-4)^2} = 0$

$\Rightarrow x = 0$ (and $x = \pm 2$)

$f''(x) = \frac{8(3x^2 + 4)}{(x^2-4)^3} = 0$

$\Rightarrow$ never (and $x, y = 0$)



① Domain: $x \neq \pm 2$

② Asymptotes: $x = \pm 2$, $y = 1$ ($\lim_{x \to \infty} f(x) = 1$)

③ Critical: $x = \pm 2, x = 0$

Poss. infl. $x = \pm 2$

④ Values: $f(0) = 0$

$f(\pm 2)$ undefined