

Panel 1

## Local Extrema, Inflection Points, and the like

Deriv Theorem: If  $f$  has a local extrema at  $x=c$ ,  
then  $f'(c)=0$  or  $f'$  does not exist (critical)

If  $f'(x) > 0$ :  $f$  up  $\nearrow$

$f'(x) < 0$ :  $f$  down  $\searrow$

$f''(x) > 0$ :  $f$  concave up  $\smile$

$f''(x) < 0$ :  $f$  concave down  $\frown$

Def: If  $f'(c)=0$  or  $f'(c)$  d.n.e.  $\rightarrow x$  is critical.

If  $f''(c)=0$  or  $f''(c)$  d.n.e.  $\rightarrow x$  is possible inflection point.

Panel 2

To discuss local extrema,  
increasing, or decreasing

① Find  $f'$

② Critical points, i.e.  
 $f'=0$  or  $f'$  d.n.e.

③ Make "sign table" for  $f'$ ,  $f$

$f'$	+	-	+
$f$	$\nearrow$	$\searrow$	$\nearrow$

from ②

To discuss inflection points  
and concavity

① Find  $f''$

② Solve  $f''(x)=0$  or  
 $f''(x)$  d.n.e.

③ Make "sign table" for  $f''$ ,  $f$

$f''$	-	+	-
$f$	$\cap$	$\cup$	$\cap$

from ②

Panel 3

① Find all rel. extrema and intervals where  $f$  is increasing/decreasing for  $f(x) = 3x^4 + 4x^3 - 12x^2$

②  $f'(x) = 12x^3 + 12x^2 - 24x = 0$

③  $12x(x^2 + x - 2) = 0 \Rightarrow x = 0$   
 $12x(x+2)(x-1) = 0 \Rightarrow x = 1, -2$  } critical

④

	$x < -2$	$-2 < x < 0$	$0 < x < 1$	$x > 1$	
$f'$	-	+	=	+	
	↘	↗	↘	↗	

$x = 0$  is max  
 $x = -2$  min  
 $x = 1$  min

Panel 4

Find all inflection points and intervals where  $f$  is concave up or down for  $f(x) = x^4 - 6x^2$

①  $f'(x) = 4x^3 - 12x = 4x(x^2 - 3)$   
 $f''(x) = 12x^2 - 12$

②  $f''(x) = 0 \Leftrightarrow 12(x^2 - 1) = 0 \Rightarrow x = \pm 1$

③

	$x < -1$	$-1 < x < 1$	$x > 1$	
$f''$	+	-	+	
$f$	∪	∩	∪	✓

Panel 5

## Curve Sketching

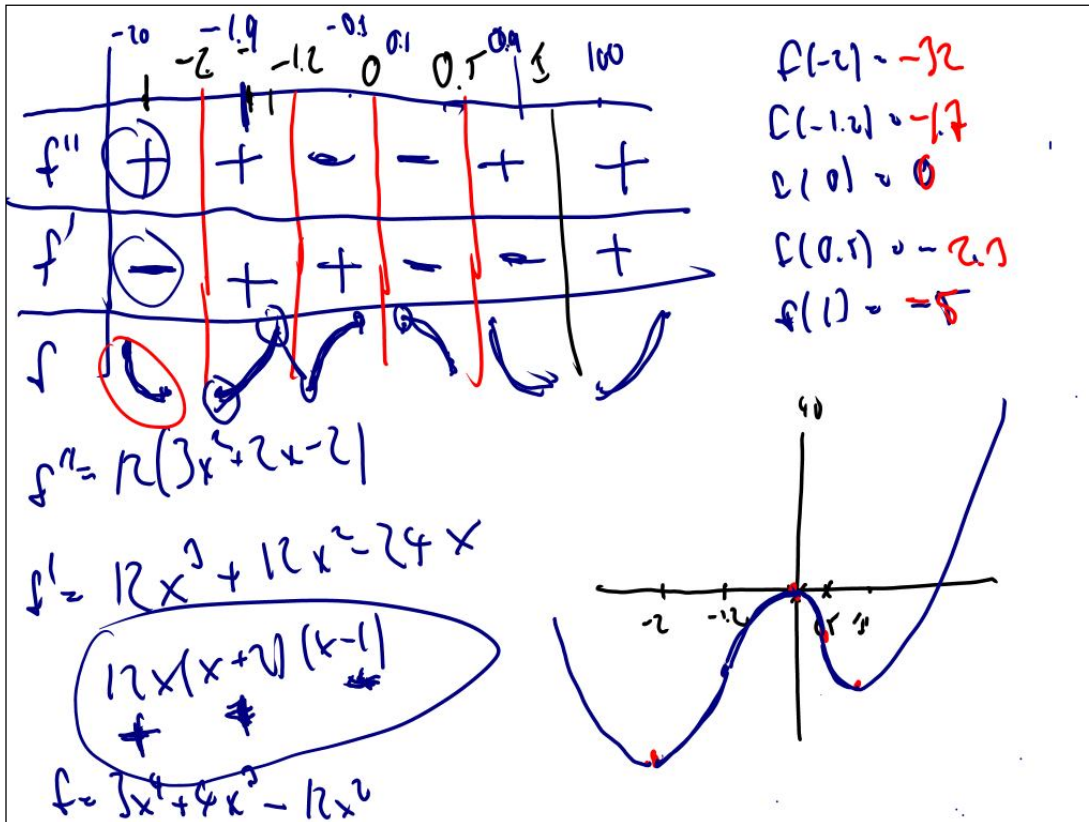
- ① Find domain
- ② Find y-intercept
- ③ Find horiz. and vert. asymptotes
- ~~④ Find critical pts~~
- ⑤ Find critical points ( $f' = 0$  or d.n.e.)
- ⑥ Find possible inflection points ( $f'' = 0$  or d.n.e.)
- ⑦ create THE TABLE with +/- values
- ⑧ create THE VALUES  $f(\text{special points})$
- ⑨ Sketch the graph

Panel 6

Ex. Sketch the graph of  $f(x) = 3x^4 + 4x^3 - 12x^2$

- ① Domain
  - ② Intercepts (y)
  - ③ Asymptotes
  - ④ Critical pts
  - ⑤ poss. inf. pts
  - ⑥ the Table
  - ⑦ the Values
  - ⑧ the Graph
- ①  $D = \mathbb{R}$
  - ② Find  $f(0) = 0 \rightarrow (0,0) \rightarrow$  y-int.
  - ③ None
  - ④  $f'(x) = 12x^3 + 12x^2 - 24x = 12x(x^2 + x - 2) = 0$   
 $x = 0, x = -2, 1$   
 critical  $-2, 1$
  - ⑤  $f''(x) = 36x^2 + 24x - 24 = -12(3x^2 + 2x - 2) = 0$   
 $ax^2 + bx + c$   
 $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 + 24}}{6}$   
 $= \frac{-2 \pm 5.29}{6} = \begin{cases} -2.29\% = -1.2156 \\ 3.29\% = 0.549 \end{cases}$

Panel 7



Panel 8

Remember Asymptotes

Horizontal asympt.:  $\lim_{x \rightarrow \pm\infty} f(x) = y$

Vertical asympt.:  $\lim_{x \rightarrow c^{\pm}} f(x) = \infty$

Ex:  $f(x) = \frac{3x^2}{x^2-1}$   
 Horiz:  $y = \lim_{x \rightarrow \pm\infty} \frac{3x^2}{x^2-1} = \frac{3}{1} = 3$   
 Vert:  $\lim_{x \rightarrow 1^+} \frac{3x^2}{x^2-1} = +\infty$ ,  $\lim_{x \rightarrow 1^-} \frac{3x^2}{x^2-1} = -\infty$   
 $\lim_{x \rightarrow -1^+} \frac{3x^2}{x^2-1} = -\infty$ ,  $\lim_{x \rightarrow -1^-} \frac{3x^2}{x^2-1} = +\infty$

Panel 9

Sketch  $f(x) = \frac{2x^2}{x^2-1}$

HA.  $f'(x) = \frac{-4x}{(x^2-1)^2}$ ,  $f''(x) = \frac{12x^2+4}{(x^2-1)^3}$

①  $x \neq \pm 1$   
 ②  $f(0) = 0$   
 ③  $g = \lim_{x \rightarrow \infty} \frac{2x^2}{x^2} = 2$   
 $x = 1, x = -1$   
 ④  $x = 0, x = \pm 1$   
 ⑤ none,  $x = \pm 1, -$

① Domain ✓  
 ② Intercepts  
 ③ Asymptotes  
 ④ Critical pts  
 ⑤ poss. inf. pts  
 ⑥ the Table  
 ⑦ the Values  
 ⑧ the Graph

$f''$	+	-	-	+
$f'$	+	+	-	-
$f$				

Panel 10

$f(x) = \frac{x}{x^3-2}$

$f'(x) = \frac{1(x^3-2) - x \cdot 3x^2}{(x^3-2)^2} = \frac{x^3-2-3x^3}{(x^3-2)^2} = \frac{-2(x^3+1)}{(x^3-2)^2}$

$f''(x) = \frac{-6x^2(x^3-2)^{-3} + 2(x^3+1) \cdot 2(x^3-2)^{-4} \cdot 3x^2}{(x^3-2)^4}$

$= \frac{-6x^2(x^3-2) + 12x^2(x^3+1)}{(x^3-2)^3} = \frac{-6x^5 + 12x^2 + 12x^5 + 12x^2}{(x^3-2)^3} = \frac{6x^5 + 24x^2}{(x^3-2)^3}$

$= \frac{6x^2(x^3+4)}{(x^3-2)^3}$

Panel 11

Sketch  $f(x) = \frac{x}{x^3-2} \Rightarrow f'(x) = \frac{-2(x^3+1)}{(x^3-2)^2}$

$f''(x) = \frac{6x^2(x^3+4)}{(x^3-2)^3}$

①  $x \neq \sqrt[3]{2}$   
 ②  $f(0) = 0$   
 ③  $y = 0$  and  $x = \sqrt[3]{2}$   
 ④  $x = -1$   
 ⑤  $x = 0, x = -\sqrt[3]{4}$   
 ⑥  $-10, -\sqrt[3]{4}, -1, 0, \sqrt[3]{2}, 10$

⑦ Domain  
 ⑧ Intercepts  
 ⑨ Asymptotes  
 ⑩ Critical pts  
 ⑪ poss. inf. pts  
 ⑫ the Table  
 ⑬ the Values  
 ⑭ the Graph

$f'$	+	+	-	-	-
$f''$	+	-	-	-	+
$f$					

⑦  $f(-\sqrt[3]{4}) = 0.264$   
 $f(-1) = 1/3$   
 $f(0) = 0$

Panel 12

