Panel 1

Local Extremes Inflection Points, and the like

Basic Theorem: If f has a local extremum at $x=c$,
then $f'(c)=0$ or f' does not exist (critical)

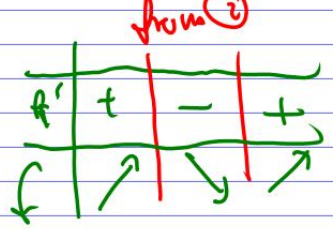
If $f'(x)>0$: f up ↗
$f'(x)<0$: f down ↘
$f''(x)>0$: f concave up ☺
$f''(x)<0$: f concave down ☹

Def: If $f'(c)=0$ or $f'(c)$ d.n.e. → x is critical.
If $f''(c)=0$ or $f''(c)$ d.n.e. → x is possible inflection point.

Panel 2

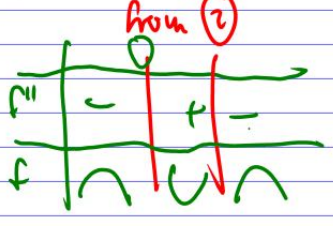
To discuss local extrema, increasing, or decreasing

1. Find f'
2. Critical points, i.e. $f'=0$ or f' d.n.e.
3. Make "sign table" for f', f



To discuss inflection points and concavity

1. Find f''
2. Solve $f''(x)=0$ or $f''(x)$ d.n.e.
3. Make "sign table" for f'', f



Panel 3

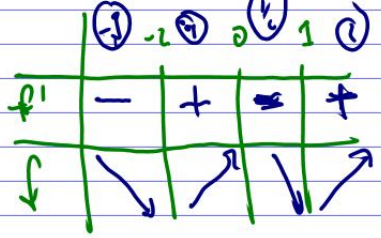
(1) Find all rel. extrema and intervals where f is increasing / decreasing for $f(x) = 3x^4 + 4x^3 - 12x^2$

① $f'(x) = 12x^3 + 12x^2 - 24x = 0$

② $12x(x^2 + x - 2) = 0 \Rightarrow x = 0$

$12x(x+2)(x-1) = 0 \Rightarrow x = 1, -2$ } critical

③



$x = 0$ is max

$x = -2$ min

$x = 1$ min

Panel 4

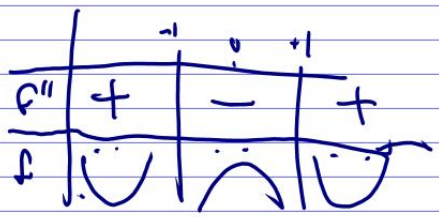
Find all inflection points and intervals where f is concave up or down for $f(x) = x^4 - 6x^2$

① $f'(x) = 4x^3 - 12x = 4x(x^2 - 3)$

$f''(x) = 12x^2 - 12$

② $f''(x) = 0 \Leftrightarrow 12(x^2 - 1) = 0 \Rightarrow x = \pm 1$

③



✓

Panel 5

Curve Sketching

1. Find domain
2. Find y-intercept
3. Find horiz. and vert. asymptotes
4. ~~Find critical pts~~
5. Find critical points ($f'=0$ or d.n.e.)
6. Find possible inflection point ($f''=0$ or d.n.e.)
7. Create THE TABLE with +/− values
8. Create THE VALUES f(special points)
9. Sketch the graph

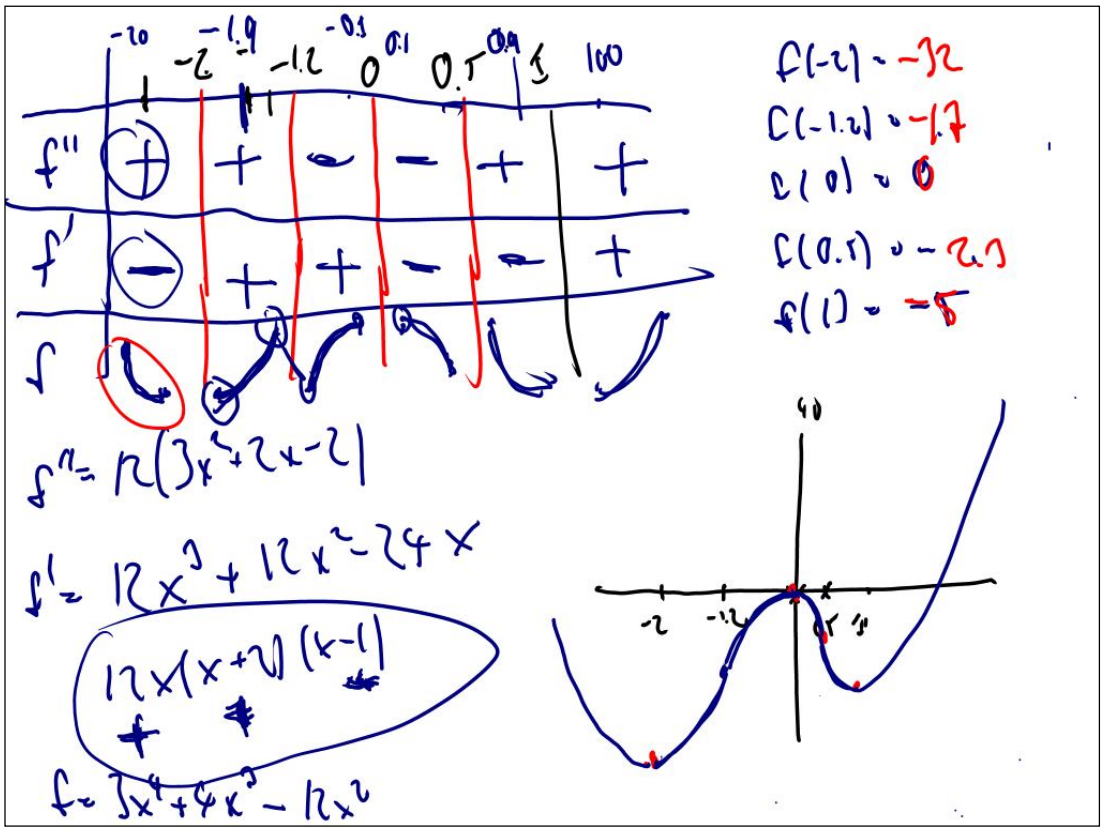
Panel 6

Ex: Sketch the graph of $f(x)=3x^4+4x^3-12x^2$

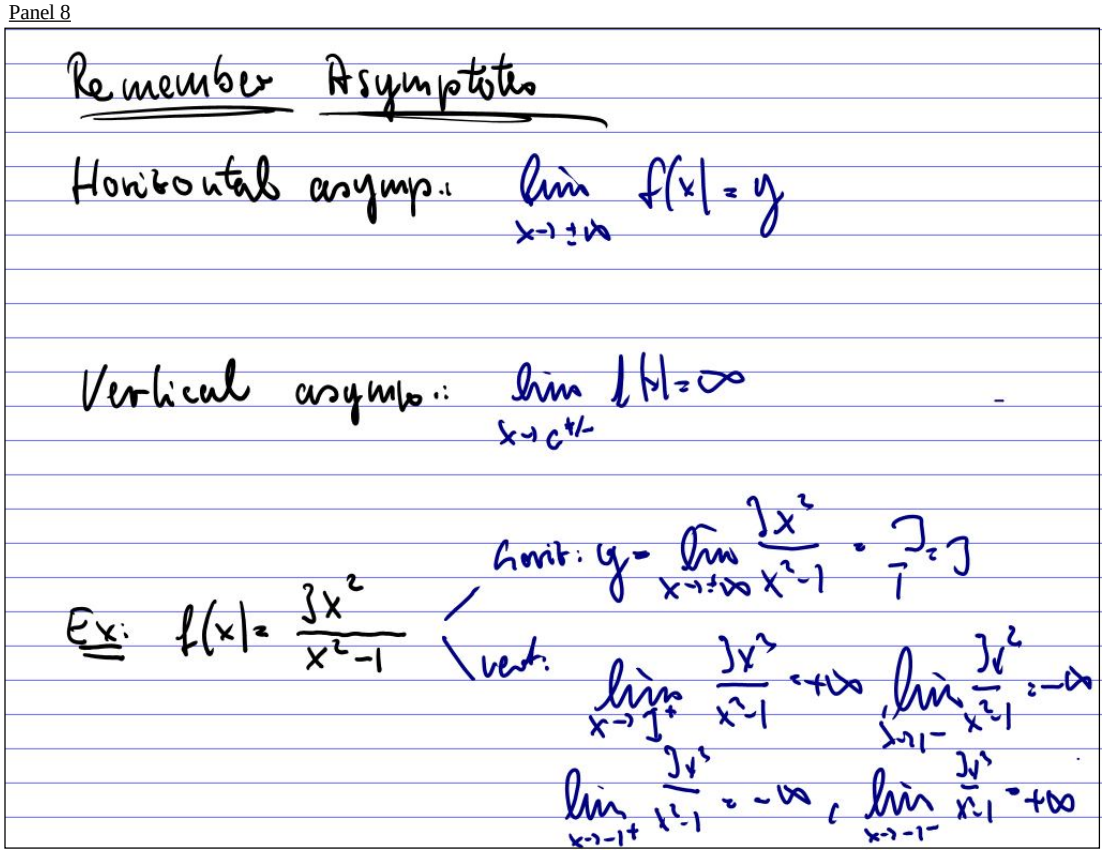
1. $D=\mathbb{R}$
2. Find $f(0)=0 \Rightarrow (0,0)$ is y-int.
3. None
4. $f'(x)=12x^3+12x^2-24x=$ $x=0,\ x=-2,1$ critical $-2, 1$
   $12x(x^2+x-2)=0$
5. $f''(x)=36x^2+24x-24=$
   $-12(3x^2+2x-2)=0$ ax^2+bx+c
   $x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{4+24}}{6}$
   $=\frac{-2\pm 5.29}{6}=$ -1.215, 0.715

1. Domain
2. Intercepts (y)
3. Asymptotes
4. Critical pts
5. poss. infl. pts
6. the Table
7. the Values
8. the Graph

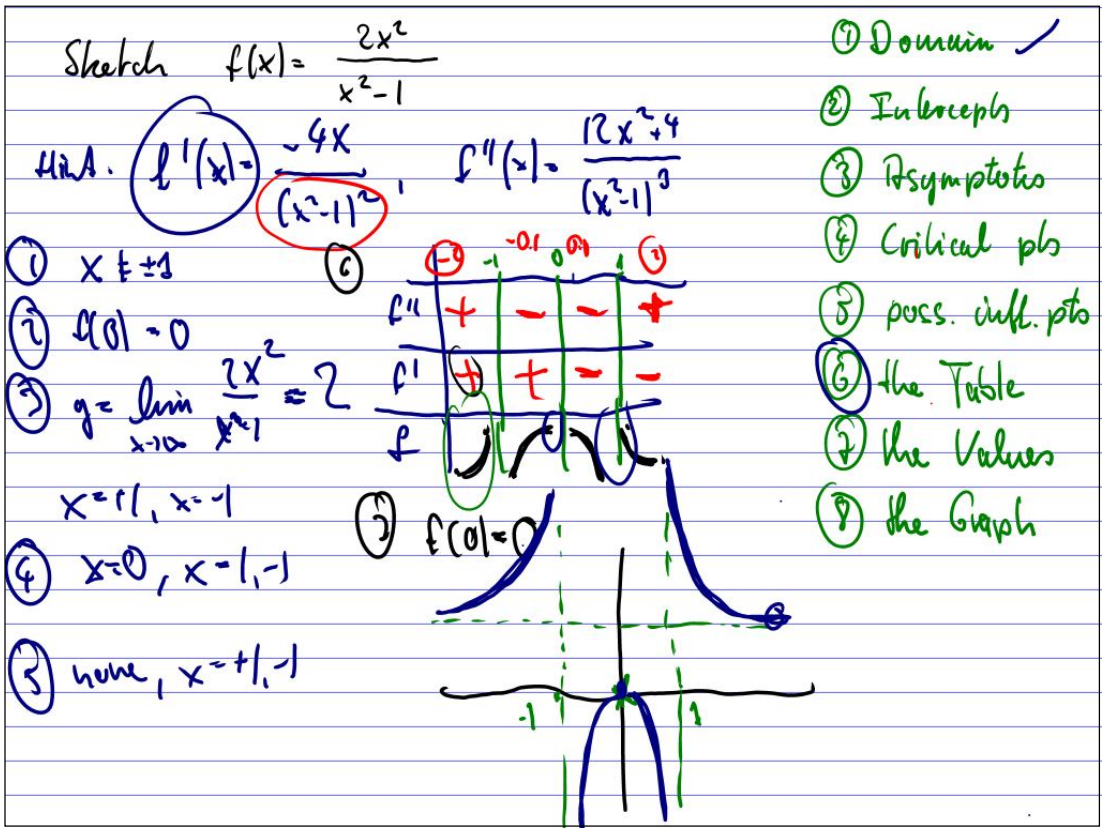
Panel 7



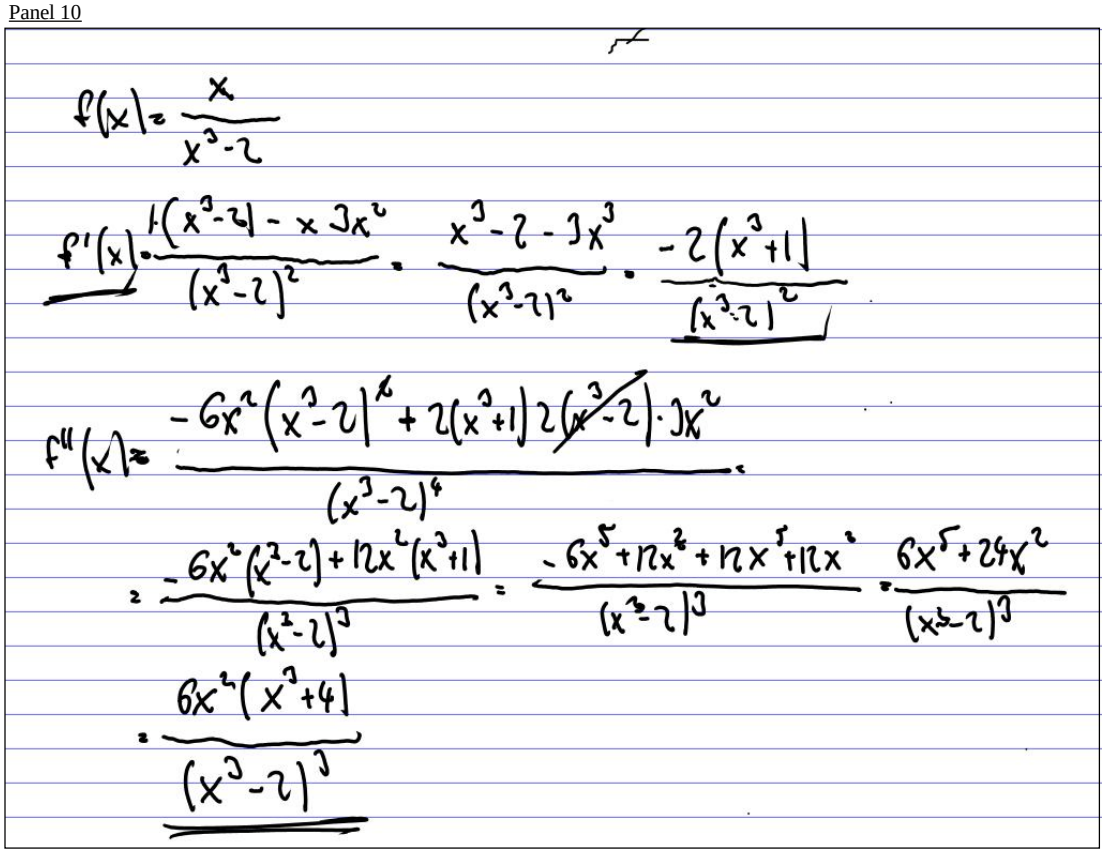
Panel 8



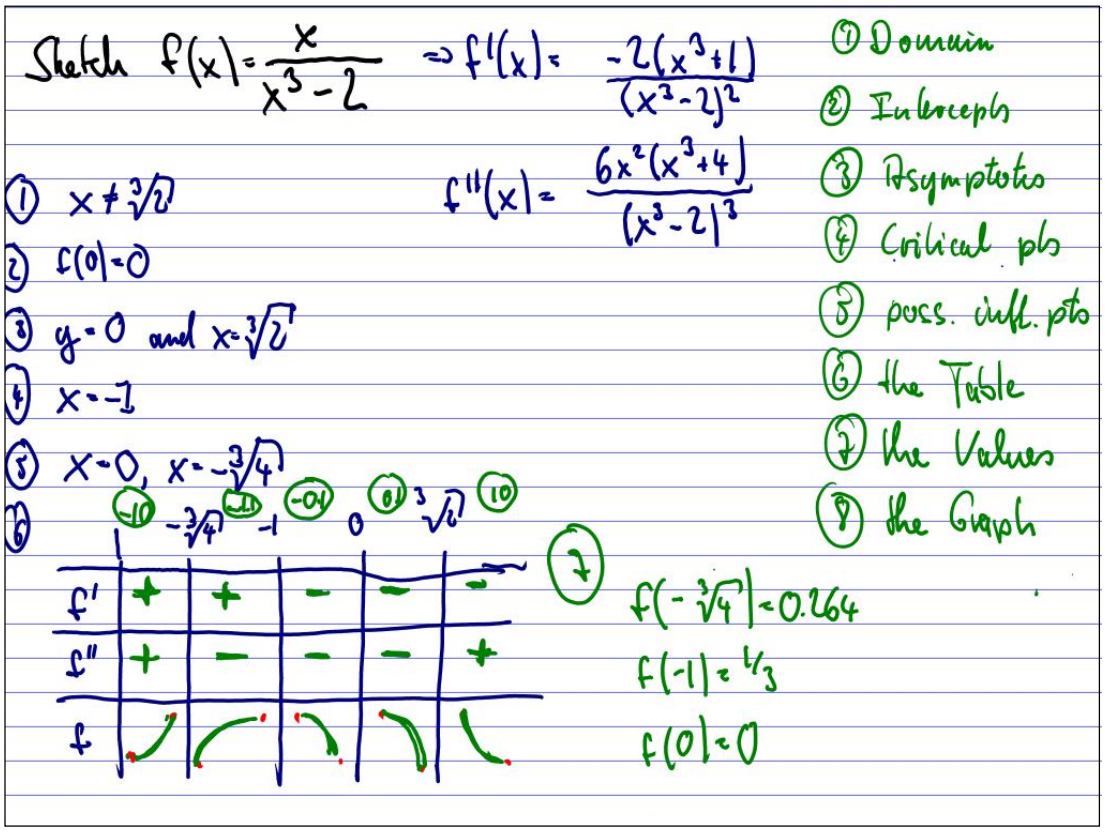
Panel 9



Panel 10



Panel 11



Panel 12