Panel 1

$$\lim_{x\to 2}\frac{4-x^2}{x-2} = \lim_{x\to 2}\frac{-\cancel{(2-x)}(2+x)}{\cancel{(x-2)}} = -4$$

$$\lim_{x\to 0^+}\frac{x-1}{x} = \frac{-1}{0^+} \quad \cancel{\text{undef.}} \quad -1 \quad = -\infty$$

$$\lim_{x\to 0}\frac{\cos(x)-1}{x} = 0$$

$$\lim_{x\to 0^-}\frac{x-1}{x} = +\infty$$

$$\lim_{x\to 0}\frac{x-1}{x} \quad \text{d.n.e.}$$

$$\lim_{x\to 0}\frac{\tan(x)}{x} = \lim_{x\to 0}\frac{\frac{\sin(x)}{\cos(x)}}{x} =$$

$$\lim_{x\to 0}\frac{\sin(x)}{\cos(x)}\cdot\frac{1}{x} = \lim_{x\to 0}\frac{\sin(x)}{x}\cdot\frac{1}{\cos(x)}$$

$\to 1$ $\quad 1$

Panel 2

$$f(x) = \begin{cases} x^2 & x \geq 1 \\ 2-x & x < 1 \end{cases}$$

Cont. at $x=0$?

$x=1$?

$$2 = \lim_{x\to 0} f(x) = f(0) ? = 2 \qquad \text{yes!}$$

$$\lim_{x\to 1} f(x) = 1 \quad \begin{cases} \lim_{x\to 1^+} f(x) = 1^2 \\ \lim_{x\to 1^-} f(x) = 2-1 = 1 \end{cases} \quad f(1) = 1^2 = 1$$

Panel 3

#9) $d(t) = 3t^4 - 4t^3$

avg. speed between 2 and 4 : $\frac{d(4)-d(2)}{4-2}$

inst. velocity : $d'(3)$: $d'(x) = \underline{12t^3 - 12t^2}$
at $t=3$

acceleration: $a = v'(t) = d''(x) = 36t^2 - 24t$
at $t=3$. $a(1) > 0$

rest ? $v(t) = 0 = 12t^3 - 12t^2 = 0$
$12t^2(t-1) = 0$
$t = 0, 1$ at Rest!

Panel 4

Last Time

Chain Rule: $\frac{d}{dx} f(g(x))$

Ex: $f(x) = (3x^2 - 1)^{10}$

Local max/min

① Find f'

② Find critical points

③ Setup table for signs of f'

Panel 5

$$f(x) = 4x^2 + \frac{3}{2\sqrt{x^3}} + \frac{2}{x^3} - \sin(\pi^2)$$ ¶

$$f(x) = 4x^2 + \frac{3}{2} x^{-3/2} + 2x^{-3} - \sin(\pi^2)$$

$$f'(x) = 8x + \left(\frac{3}{2}\right)\left(-\frac{3}{2}\right) x^{-5/2} + 2(-3)x^{-4} + 0$$

Panel 6

Ex: Rel. extrema of $f(x) = x^{1/3}(2-x)$ HW

(1) $f'(x) = \frac{1}{3}x^{-2/3}(2-x) + x^{1/3}(-1)$

$$f'(x) = x^{-2/3}\left(\frac{1}{3}(2-x) - x^{1}\right) = x^{-2/3}\left(\frac{2}{3} - \frac{1}{3}x - x\right) =$$

$$= x^{-2/3}\left(\frac{2}{3} - \frac{4}{3}x\right) =$$

$$= \frac{1}{3}x^{-2/3}(2-4x) = 0$$

$x = 0, \frac{1}{2}$ 0.0000001 10000000

	-1	0	1/2
f'	+	+	−
f	↗	↗	↘

$x = \frac{1}{2}$ is max! ✓

?

last time

Panel 7

$x^{-2/3} \cdot x^{1} = x^{1/3}$

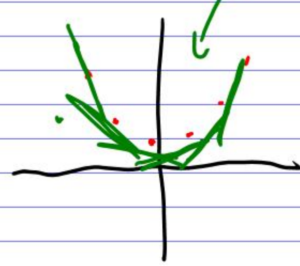
Panel 8

What does f'' say about f?

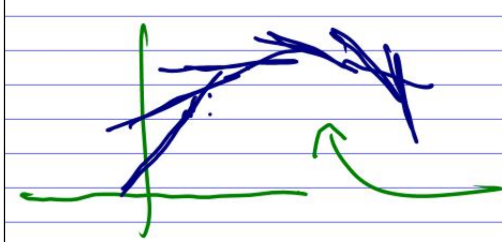
f' says if f is incr. or decr.

$f'' > 0 \Rightarrow f'$ is increasing

concave up



$f'' < 0 \Rightarrow f'$ is decreasing



concave down!

Panel 9

<u>Concavity:</u>

If $f''(x) > 0$ then f is concave up

If $f''(x) < 0$ then f is concave down

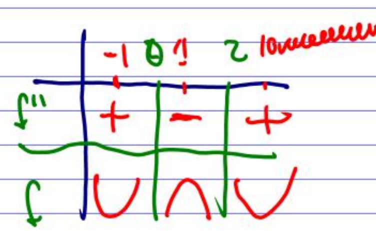
If $f''(c) = 0$ and f changes concavity at $x = c$:

<u>Ex:</u> $f(x) = x^4 - 4x^3$ - discuss concavity

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x = 0$

$\boxed{12x(x-2)} = 0$



$x = 0, 2$ are possible points of inflection, i.e. ~~critical~~ concavity changes!

Panel 10

<u>Ex:</u> $f(x) = x^4 - 2x^2 + 3$ Discuss inc / dec / concavity

$f'(x) = 4x^3 - 4x = 4x(x^2 - 1)$ f' f''

$= 4x(x^2-1) = 0 \Rightarrow x = 0, 1, -1$ ← critical

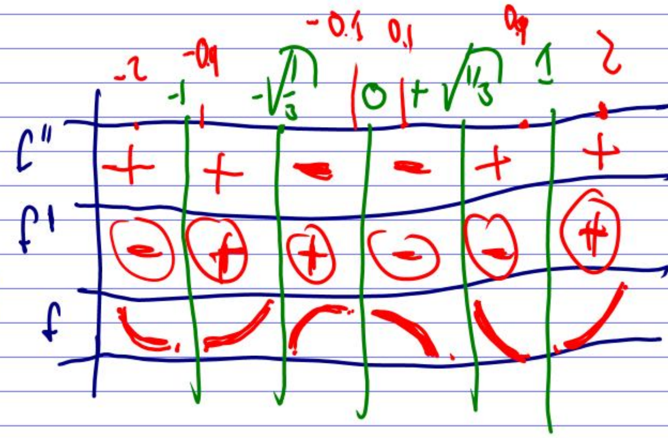
$f''(x) = 12x^2 - 4 = 0$

$= 4(3x^2 - 1) = 0$

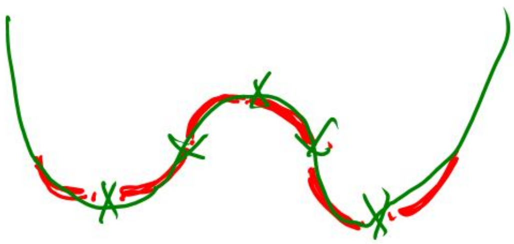
$3x^2 - 1 = 0$

$3x^2 = 1$

$x = \pm\sqrt{\frac{1}{3}}$



Panel 11



Panel 12

Ex: $f(x) = x^{2/3}(6-x)^{1/3}$ Discuss inc/dec/concavity

$$f'(x) = \frac{2}{3}x^{-1/3}(6-x)^{1/3} + x^{2/3}\frac{1}{3}(6-x)^{-2/3}(-1)$$

Hint: $f'(x) = \frac{4-x}{x^{1/3}(6-x)^{2/3}}$ $\qquad f''(x) = \frac{-8}{x^{4/3}(6-x)^{5/3}}$

$f' = 0$: $x = 4$

f' undef: $x = 0, 6$ are critical

$f'' = 0$: none

f'' undef : $x = 0, 6$

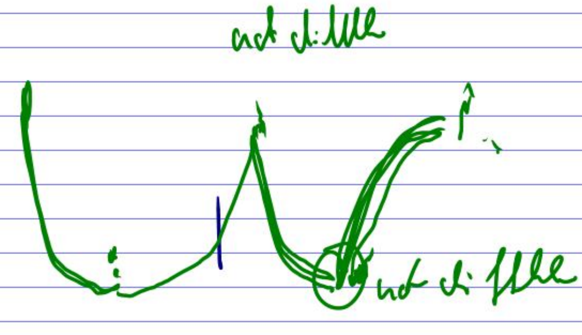
increasing

up down

	-1	0	1	4	5	6	10
f''	+		+		+		−
f'	−		+		−		+

decr.

Panel 13



Panel 14

Local Extrema, Inflection Points, and the like

Basic Theorem: If f has a local extrema at $x=c$,
then $f'(c)=0$ or $f'(c)$ does not exist.

If $f'(x)>0$: f is increasing
$f'(x)<0$: f is decreasing
$f''(x)>0$: f is concave up
$f''(x)<0$: f is concave down

Def: If $f'(c)=0$ or $f'(c)$ d.n.e. $\rightarrow$ x is critical pt.
If $f''(c)=0$ or $f''(c)$ d.n.e. $\rightarrow$ x is possible infl. pt.

Panel 15

Curve Sketching

1. Find domain
2. Find y-intercept
3. Find horiz. and vert. asymptotes

5. Find critical points ($f'=0$ or d.n.e.)
6. Find possible inflection point ($f''=0$ or d.n.e)
7. Create THE TABLE
8. Create THE VALUES
9. Sketch the graph

Panel 16

Ex: Sketch the graph of $p(x) = x^6 + 6x^5 + 18x^3$

$p'(x) = 6x^5 + 30x^4 + 54x^2$
$= 6x^2(x^3 + 5x^2 + 9) = 0$
$x = 0, -5.38$

$p''(x) = 30x^4 + 120x^3 + 108x =$
$= x(30x^3 + 120x^2 + 108) = 0$
$x = 0, -4.2$

	-1000	-5.3 -5	-4.2 -1	0 1000
f''	-			+
f'				
f				

1. Domain: all x
2. y-int: set $x=0$, $y=0$
3. Asymptotes: none
4. critical
5. possible infl

THE table

THE values

HW: Fill in rest