Panel 1

Last Time

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

Ex: $f(x) = (3x^2-1)^{10}$ $f'(x) = 10(3x^2-1)^9 \cdot 6x$

Local max/min

1. Find f'
2. Find critical points: $f'(x)=0$ or $f'(x)$ d.n.e.
3. Setup table for signs of f'

Private Freehand 1

Panel 2

Ex: Find local extrema for $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$
and identify all intervals where f is increasing.

① $f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2) = 12x(x-2)(x+1)$

② Critical points: $f'(x) = 0 \Rightarrow \underline{x = 0, 2, -1}$

~~$f'(x)$ d.n.e~~ None

	-2	-1	$-\frac{1}{2}$	0	1	2	3
f'	$-$		$+$		$-$		$+$
f	↘		↗		↘		↗

$x = -1$: local min

$x = 0$: local max

$x = 2$: local min

Panel 3

$f(x) = \tan(x^3)\sqrt{3-4x} = \tan(x^3)(3-4x)^{\frac{1}{2}}$

$f'(x) = (\sec^2(x^3) \cdot 3x^2)(3-4x)^{\frac{1}{2}} + \tan(x^3)\left[\frac{1}{2}(3-4x)^{-\frac{1}{2}} \cdot (-4)\right]$

$$\left(\frac{(x^4-2x)^3+3}{((1-x)^2}\right)' = \frac{3(x^4-2x)^2(4x^3-2)(1-x)^2 - ((x^4-2x)^3+3)2(1-x)(}{((1-x)^2)^2}$$

Panel 4

$$f(x) = \left(\left((5x+4)^2+3\right)^3+2\right)^4+1$$

$$f' = 4\left(\left((5x+4)^2+3\right)^3+2\right)^3 \cdot 3\left((5x+4)^2+3\right)^2 \cdot 2(5x+4)\cdot 5$$

$$f = (2x-5)^4$$

$$f' = 4(2x-5)^3 \cdot 2 = 8(2x-5)^3$$

$$f'' = 8\cdot 3(2x-5)^2 \cdot 2 = 48(2x-5)^2$$

$$f''' = 48\cdot 2(2x-5)\cdot 2 = 192(2x-5)$$

Panel 5

$$f(x) = \frac{1}{1+\frac{1}{1+\frac{1}{(1+x)}}} = \frac{1}{1+\frac{1}{1+(1+x)^{-1}}} = \frac{1}{1+\left(1+(1+x)^{-1}\right)^{-1}}$$

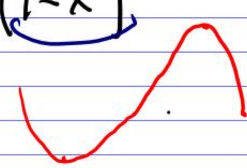
$$= \left(1+\left(1+(1+x)^{-1}\right)^{-1}\right)^{-1}$$

$$f' = (-1)\left(1+\left(1+(1+x)^{-1}\right)^{-1}\right)^{-2}(-1)\left(1+(1+x)^{-1}\right)^{-2}(-1)(1+x)^{-2}$$

Panel 6

Panel 7

Ex: Find relative extrema of $f(x) = x^3 \cdot (1-x^2)$



① $f'(x) = 3x^2(1-x^2) + x^3(-2x) =$

$= 3x^2 - 3x^4 - 2x^4 = 3x^2 - 5x^4 = 0$

$= x^2(3-5x^2) = 0$ $\quad 3 = 5x^2$

② Critical: $x=0$, $x = \pm\sqrt{\frac{3}{5}}$ $\quad \frac{3}{5} = x^2$

$-1000 \quad -\sqrt{\frac{3}{5}} \quad -0.0001 \quad 0 \quad +0.0001 \quad \sqrt{\frac{3}{5}} \quad 1000$

f'	$-$	$+$	$+$	$-$
f	↘	↗	↗	↘

$x = -\sqrt{\frac{3}{5}}$ is min

$x = +\sqrt{\frac{3}{5}}$ is max

$x = 0$ is nothing!

Panel 8

Ex: Rel. extrema of $f(x) = x^{1/3}(2-x)$ HW / Ar 7?

① $f'(x) = \frac{1}{3}x^{-2/3}(2-x) + x^{1/3}(-1)$

$f'(x) = x^{-2/3}\left(\frac{1}{3}(2-x) - x^1\right) = x^{-2/3}\left(\frac{2}{3} - \frac{1}{3}x - x\right) =$

$= x^{-2/3}\left(\frac{2}{3} - \frac{4}{3}x\right) =$

$= \frac{1}{3}x^{-2/3}(2-4x) = 0$

$x = 0,\ 1/2$

0.0000000001 1000000000

	-1	0	$1/2$
f'	+	+	−
f	↗	↗	↘

$x = 1/2$ is max! ✓

?

Panel 9

We have seen f' to check

⇒ look at f'': what does it tell you about f?

What is it good for?

Wal

Quiz ⇒ Sound of $\frac{df}{dx}$

local max/min