Panel 1

Differentiation

Power Rule $\frac{d}{dx} x^p = p \cdot x^{p-1}$ ✓

Product Rule $\frac{d}{dx} \left(f(x) \cdot g(x)\right) = \left(f'(x)\right) g(x) + f(x) \left(g'(x)\right)$

Quotient Rule $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{\left(f'(x)\right) g(x) - f(x) \left(g'(x)\right)}{[g(x)]^2}$

no change

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

composition

Panel 2

Chain Rule Examples

$f(x) = \cancel{\sin}(\cancel{x^2}) \Rightarrow f'(x) = \cos(x^2) \cdot 2 \cdot x$

$f(x) = \sqrt{1-2x^2} = (1-2x^2)^{1/2} \quad f'(x) = \frac{1}{2}(1-2x^2)^{-1/2}(-4x)$

$f(x) = \cancel{\sin}(\cancel{\cos}(x^3)) \quad f'(x) = \cos(\cos(x^3))(-\sin(x^3))3x^2$

$f(x) = \cancel{\sin}\left(\frac{x}{1-x}\right) \quad f'(x) = \cos\left(\frac{x}{1-x}\right) \cdot \left(\frac{(1)(1-x) - x(-1)}{(1-x)^2}\right)$

$f(x) = \frac{\sin(x)}{\sin(1-x)} \quad f'(x) = \frac{(\cos(x))\sin(1-x) - \sin(x) \cdot (\cos(1-x)(-1))}{\sin^2(1-x)}$ ✓

Panel 3

$f(x) = x^{1/3} \Rightarrow f'(x) = \frac{1}{3} x^{-2/3}$

$f(x) = x^3 \Rightarrow f'(x) = 3x^2$

Panel 4

$f(x) = \frac{x^2 \sin(x)}{\cos(x-2x^2)}$

$f'(x) = \frac{((2x)\sin(x) + x^2\cos(x))\cos(x-2x^2) - x^2\sin(x)(-\sin(x-2x^2))(1-4x)}{[\cos(x-2x^2)]^2}$

$h(x) = \left(\frac{x\sin(x^2)}{\cos(x^5)}\right)^3$

$h'(x) = 3\left(\frac{x\sin(x^2)}{\cos(x^5)}\right)^2 \left(\frac{(\sin(x^2) + x\cdot\cos(x^2)2x)\cos(x^5) - x\sin(x^2)(\ldots)}{\cos^2(x^5)}\right)$

$-\sin(x^5)\cdot 5x^4$

$j(x) = \sqrt{\sin(\tan(x^2))}$

foo Lowes

Panel 5

Quotient Rule: not necessary.

Ex: $f(x) = \frac{x^2-3}{x^2-2x+1} \Rightarrow f'(x) = \frac{(2x)(x^2-2x+1)-(x^2-3)(2x-2)}{(x^2-2x+1)^2}$

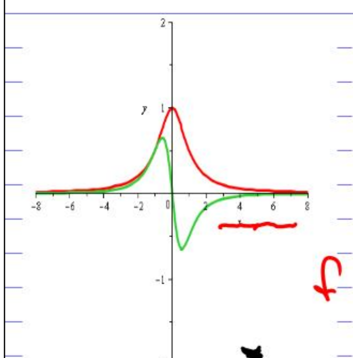
$f(x) = \frac{x^2-3}{x^2-2x+1} = (x^2-3)\cdot\frac{1}{x^2-2x+1} = (x^2-3)(x^2-2x+1)^{-1}$

$f'(x) = 2x(x^2-2x+1)^{-1} + (x^2-1)\cdot(-1)(x^2-2x+1)^{-2}\cdot(2x-2) =$ equal

In principle, replace quotient rule by chain + product.
Most people don't!!

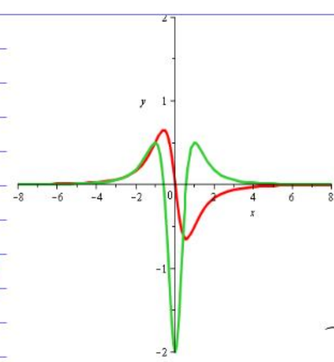
Panel 6

Identify f and f'



f

f'



f'

f

Panel 7

Applications of Derivatives

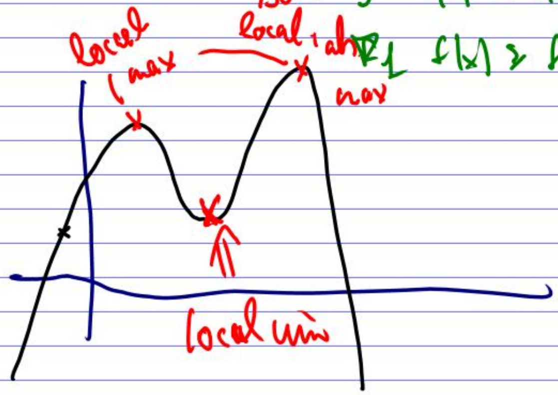
① Finding max/min

Def. If $f(x) \leq f(x_0)$, then x_0 is absolute max

If $f(x) \geq f(x_0)$, then x_0 is absolute min

Soll $\exists$ $f(x) \leq f(x_0)$ locally $\Rightarrow$ x_0 relative max

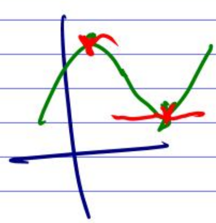
If $f(x) \geq f(x)$ locally $\Rightarrow$ x_0 relative min



Panel 8

Thm: (Fermat's Theorem)

If f has a local max or min at $x=c$ and $f'(c)$ exists, then $f'(c)=0$



Note: All points where $f'(x)=0$ or f' d.n.e. are called critical points. They could potentially be rel. max or min.

Ex $f(x)=$

Sat at 3:10

Panel 9

Recipe for finding relative extrema, i.e. max or min

Ex: $f(x) = x^3 - 3x^2 + 1$

(1) Find f'

$f'(x) = 3x^2 - 6x$

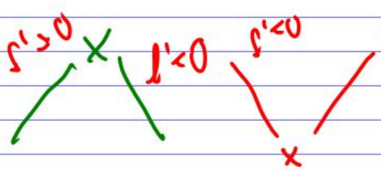
(2) $f' = 0$ and solve

$0 = 3x^2 - 6x = 3x(x-2)$

$\Rightarrow$ critical points

$\Rightarrow x = 0, 2$

(3) Check signs near critical points via table



	-1	0 (max)	1	2 (min)	3
f'	$+$		$-$		$+$
f	↗		↘		↗

Panel 10

Thm: If f is diffble and $f'(x) > 0 \Rightarrow f$ increasing

If f is diffble and $f'(x) < 0 \Rightarrow f$ decreasing

Ex: $f(x) = x^{3/5}(4-x)$ Find local extrema, i.e. max/min

$= 4x^{3/5} - x^{8/5}$

$\frac{12}{5} = \frac{8}{5}x$

$f'(x) = 4\,\frac{3}{5}x^{-2/5} - \frac{8}{5}x^{3/5} = x^{-2/5}\left(\frac{12}{5} - \frac{8}{5}x^1\right) = 0$

f' is undefined for $x = 0$

is zero for $x = \frac{12}{8} = \frac{3}{2}$

	-1	0 (none)	1	$3/2$ (max)	2
f'	$+$		$+$		$-$
f	↗		↗		↘

$\frac{12}{5} - \frac{8}{5}x = 0$

$\frac{5}{8}\,\frac{12}{5} = \frac{8}{5}x \cdot \frac{5}{8}$

Panel 11

Ex: Find local extrema for $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$
and identify all intervals where f is increasing

① $f'(x) = 12x^3 - 12x^2 - 24x$

② $0 = 12x^3 - 12x^2 - 24x =$

$= 12x(x^2 - x - 2) =$

$= 12x(x-2)(x+1)$

$\Rightarrow x = 0, 2, -1$ are critical

③ $-2 \quad -1 \quad -\tfrac{1}{2} \quad 0 \quad 1 \quad 2 \quad 3$

f'	$-$	$+$	$-$	$+$
f	↘	↗	↘	↗

Thus:

$x = -1$ is loc min

$x = 0$ is loc. max

$x = 2$ is loc. min