Panel 1

Math 1501: Last time

Differentiation: $f'(x) = \lim_{t \to x} \frac{f(x)-f(t)}{x-t}$

deriv. ≅ slope of tangent $= \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

inst. rate of change

inst. velocity (speed)

marginal revenue, cost, or profit

Shortcut rule for $f(x) = x^p$: $\Rightarrow f'(x) = p\,x^{p-1}$

Ex: $f(x) = \sqrt{x^5} = x^{5/2} \Rightarrow f'(x) = \frac{5}{2} x^{5/2 - 1} = \underline{\underline{\frac{5}{2} x^{3/2}}}$

Panel 2

$h(t) = 58t - 0.83t^2$ $\qquad h(t) = t^1 \quad h'(t) = 1 \cdot t^0 = 1$

avg. velocity between $t=0$ and $t=2$

$\frac{h(2)-h(0)}{2-0} = \frac{58 \cdot 2 - 0.83 \cdot 4 - 0}{2-0}$

avg. velocity between $t=2$ and $t=4$?

$\frac{h(4)-h(2)}{4-2} = \frac{(58 \cdot 4 - 0.83 \cdot 16) - (58 \cdot 2 - 0.83 \cdot 4)}{2}$

a) velocity at $t=1$: $\lim_{t \to 1} \frac{h(1)-h(t)}{1-t} = ?$ $\qquad$ At $t=4$: $h'(4)$

$h'(t) = 58 - 2 \cdot 0.83 \cdot t$ $\qquad h'(1) = 58 - 2 \cdot 0.83 \cdot 1$

Panel 3

Find derivative of $f(x) = 7x^2 - 2x$

a) $\lim\limits_{t \to x} \frac{f(x) - f(t)}{x - t} = \lim\limits_{t \to x} \frac{(7x^2 - 2x) - (7t^2 - 2t)}{x - t}$

$= \lim\limits_{t \to x} \frac{7(x^2 - t^2) - 2(x - t)}{x - t} =$

$= \lim\limits_{t \to x} \frac{7(x-t)(x+t) - 2(x-t)}{x - t} =$

$= \lim\limits_{t \to x} \frac{\cancel{(x-t)}(7(x+t) - 2)}{\cancel{(x-t)}} = \underline{\underline{14x - 2}}$

b) $\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

c) $f'(x) = 14x - 2$

Panel 4

$\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h} =$, $f(x) = 7x^2 - 2x$

$\lim\limits_{h \to 0} \frac{(7(x+h)^2 - 2(x+h)) - (7x^2 - 2x)}{h} =$

$\lim\limits_{h \to 0} \frac{7(x^2 + 2xh + h^2) - \cancel{2x} - 2h - \cancel{7x^2} + \cancel{2x}}{h} =$

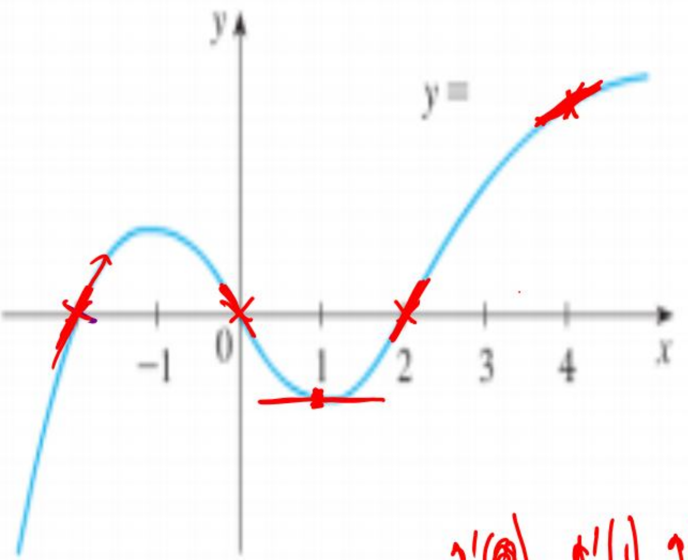
$= \lim\limits_{h \to 0} \frac{\cancel{7x^2} + 14xh + 7h^2 - \cancel{2x} - 2h - \cancel{7x^2} + \cancel{2x}}{h} =$

$\lim\limits_{h \to 0} \frac{\cancel{h}(14x + \cancel{7h}^{\to 0} - 2)}{\cancel{h}} = \underline{\underline{14x - 2}}$

Panel 5

0 $g'(-2)$ $g'(0)$ $g'(2)$ $g'(4)$ $g'(1)$

>0 <0 >0 >0 $=0$



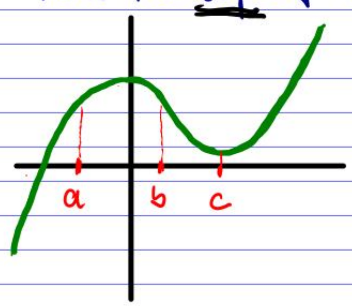
$g'(0)$, $g'(1)$, $g'(4)$, $g'(2)$, $g'(-2)$

Panel 6

Name: __________

Quiz #4

① Find the sign of the derivative f' at the given point.



a) $f'(a)$

b) $f'(b)$

c) $f'(c)$

② Use the definition of derivative to find $f'(x)$ for $f(x) = x^2 + 3$. You **must** use the definition!

Panel 7

(3) The distance traveled for a particle is $d(t) = 16t^2 + 8t$. Find

a) The average velocity between $t=1$ and $t=2$

b) The instanteneous velocity at $t=2$
(you can use any method you want)

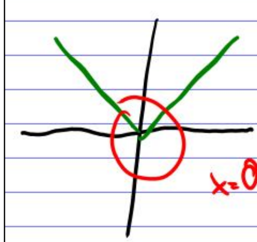
Panel 8

Not every function has a derivative.

Ex: $f(x) = |x|$.

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{|h| - 0}{h} =$$

$$= \lim_{h\to 0} \frac{|h|}{h} \ [\boxed{\text{undef}!}]$$



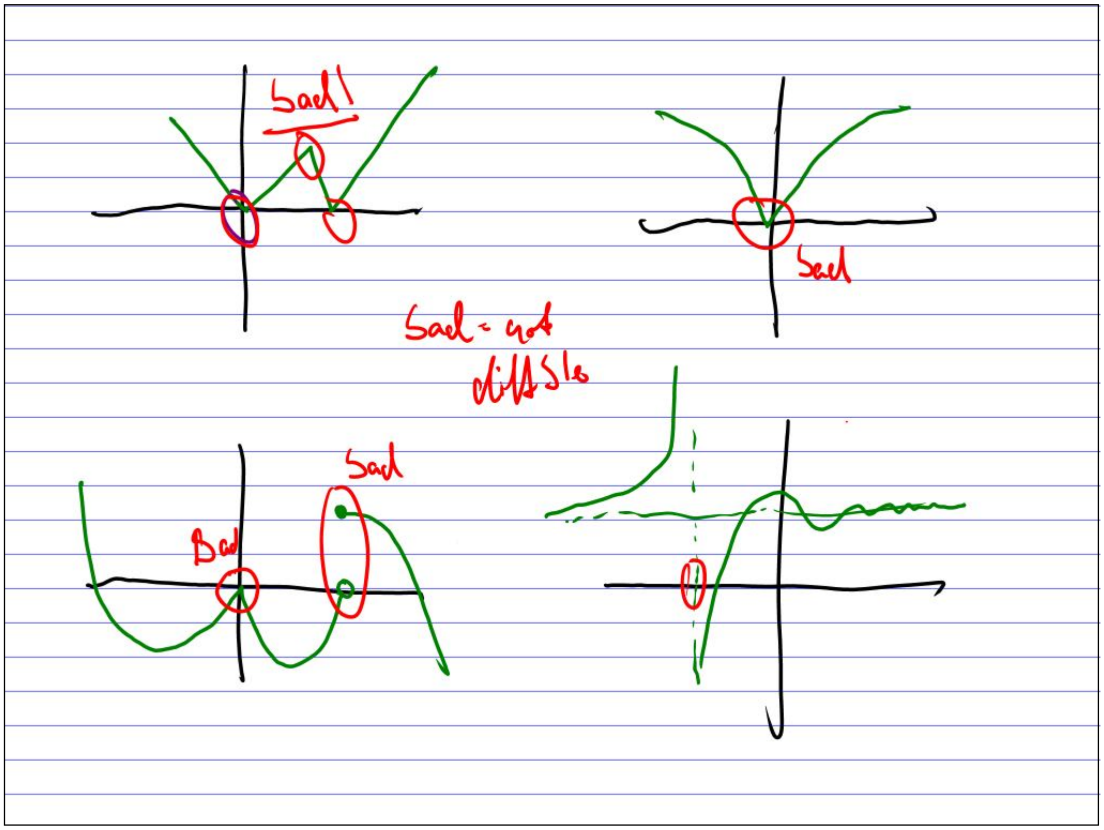
$x=0$ is "different". Suspect $f'(0)$ d.n.e

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\lim_{h\to 0^+} \frac{|h|}{h} = \lim_{h\to 0^+} \frac{\cancel{h}}{\cancel{h}} = 1$$

$$\lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}} = -1$$

Panel 9



Panel 10

Theorem: If $f(x)$ is differentiable at $x=a$ then $f(x)$ is continuous at $x=a$. The converse is false.

Proof: Cont. at $x=a$: $\lim_{x\to a} f(x) = f(a)$

$$\iff \lim_{x\to a}\left(f(x)-f(a)\right) = 0$$

$$\boxed{\lim_{x\to a}\frac{f(x)-f(a)}{(x-a)}}(x-a) =$$

$$f'(a)\cdot 0 = 0$$

If f is diff'ble then f must be cont.

$f(x)=|x|$ is cont. but not diff'ble!

Panel 11

Short cuts for differentiation:

① $f(x) = 5x^p \Rightarrow f'(x) = p \cdot x^{p-1}$ (Power Rule)

Ex: $f(x) = x^5 \Rightarrow f'(x) = 5x^4$

$f(x) = \frac{1}{x^4} = x^{-4} \Rightarrow f'(x) = -4x^{-5}$

$f(x) = \sqrt[3]{x^2} = x^{2/3} \Rightarrow \boxed{f'(x) = \frac{2}{3} x^{-1/3}} = \frac{2}{3\sqrt[3]{x}}$

Panel 12

Constant Rule

$\frac{d}{dx} c \cdot f(x) = c \cdot \frac{d}{dx} f(x)$

$(c \cdot f(x))' = c\, f'(x)$

Note: $\frac{d}{dx} f(x) = f'(x)$

Note: $\frac{d}{dx}(c) = 0$

Ex. $f(x) = 5x^2 \Rightarrow f'(x) = 5 \cdot 2x = 10x$

$f(x) = -\frac{3}{x^2} = -3x^{-2}$ $f'(x) = -3(-2)x^{-3} = 6x^{-3}$

$f(x) = \frac{5}{9x^{2/3}} = \frac{5}{9} x^{-2/3} \Rightarrow f'(x) = \frac{5}{9}\left(-\frac{2}{3}\right) x^{-5/3}$

$f(x) = 2^3 \cdot x^0 \Rightarrow f'(x) = 0$

$f'(x) = 3 \cdot 2^2$

Panel 13

Add / Subtract Rule

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

Ex: $f(x) = x^2 + x^5 - \frac{1}{x}$ (x^{-1}) $\Rightarrow f'(x) = 2x + 5x^4 + x^{-2}$

$f(x) = 3x^2 - 2\sqrt{x} + \frac{9}{x^3} - \sqrt{2}$

$f(x) = x^2(1-9x)$

$f(x) = \frac{5x^2 - 9\sqrt{x} + 3}{x}$ No can do ... can't ... $f'(x) = 5 + \frac{9}{2}x^{-3/2} - 3x^{-2}$

$= \frac{5x^2}{x} - \frac{9\sqrt{x}}{x} + \frac{3}{x} = 5x - 9x^{-1/2} + 3x^{-1}$

Panel 14

Ex: Find points for $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

Find $\boxed{y' = 0}$

$y' = 4x^3 - 12x = 0$

$\Rightarrow \underbrace{4x}\underbrace{(x^2 - 3)} = 0$

$x = 0, \; x = \sqrt{3}, \; x = -\sqrt{3}$

Panel 15

Ex: Position of particle is $f(t) = t^3 - 6t^2 + 9t$

a) velocity at time t

b) when is particle at rest?

c) when is particle moving forward?

d) acceleration at time t

a) $f'(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t-3)(t-1)$

b) at Rest for $t = 3, 1$, because $f'(t) = 0$ there

c) $f'(x) > 0$ for $t < 1$ and $t > 3$

$+ \quad - \quad +$

$0 \quad 1 \quad 2 \quad 3 \quad 4$

d) $a(t)$ is deriv of speed: $a(t) = f''(t) = 6t - 12$

Panel 16

Higher Order Derivatives

$f(x)$ is a function

$\Rightarrow$ next time!