Panel 1

Math 1401: Last time (limits at infinity)

Formal def. of limit: if $|x-a|<\delta$ then $|f(x)-L|<\varepsilon$

Squeezing Theorem: If $g(x) \leq f(x) \leq h(x)$ as $x \to a$

Special limits: $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ $\quad \lim_{x \to 0} \frac{\cos(x)-1}{x} = 0$ $\quad \lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ undef $\quad \lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = 0$

Continuity: (i) $f(a)$ exist (ii) $\lim_{x\to a} f(x)$ exists (iii) (i) = (ii)

Graphical meaning: no gaps, no holes

Types of Discontinuities: removable, jump, essential

Panel 2

Ex: Identify and classify the discontinuities

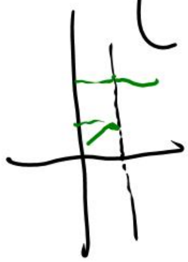
$f(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ \cancel{3}\ 1 & \text{if } x = 0 \end{cases}$ check $x = 0$: $f(0) = 3$, $\lim_{x\to 0} f(x) = 1$ Not cont. (removable)

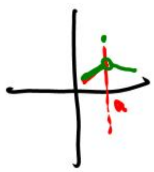
$g(x) = \begin{cases} x^2-1 & \text{if } x<1 \\ x+2 & \text{if } x \geq 1 \end{cases}$ check $x = 1$: $f(1)$ = undef. Not cont (jump

$h(x) = \begin{cases} x^2-1 & \text{if } x<0 \\ x+2 & \text{if } x>0 \end{cases}$ check $x=0$: jump

$k(x) = \begin{cases} \sin(1/x) & \text{if } x \neq 0 \\ 15 & \text{if } x = 0 \end{cases}$ essential

Panel 3





jump discontinuity at $x=a$:

(1) $\lim\limits_{x \to a^+} f(x)$ exists, $\lim\limits_{x \to a^-} f(x)$ exists $\Big]$ but not equal

(2) If $\lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to a^-} f(x)$ $\Big]$ removable

(3) else essential, i.e. $\lim\limits_{x \to a^-} f(x)$ or $\lim\limits_{x \to a^+} f(x)$ d.n.e

Panel 4

$f(x) = (x + 2x^3)^4$, $a = -1$ Continuous?

(i) $f(-1) = 3^4$

(ii) $\lim\limits_{x \to -1} f(x) = 3^4$

(iii) (i) = (ii)? ✓

$g(x) = \sqrt{x + 2x^3}$, $a = -1$

$g(-1)$ undef. $\to$ not cont

yes $\quad a = 1$

$h(x) = \sqrt{x + 2x^3} / (x-1)$ not cont. at $a = 1$

Panel 5

$$g(x) = \begin{cases} x^2 - c^2 & x < 4 \\ cx + 20 & x \geq 4 \end{cases}$$

Cont. at $x = 4$?

$g(4) = 4c + 20 = 12$

$\lim_{x\to 4} g(4) = -8 + 20 = 12$

$= 12$

$\lim_{x\to 4^-} f(x) = \underline{16 - c^2}$

$\lim_{x\to 4^+} f(x) = \underline{4c + 20}$

$16 - c^2 = 4c + 20$

$0 = c^2 + 4c + 4$

$= (c+2)^2$

$\underline{\underline{c = -2}}$

Panel 6

$$\lim_{x\to\infty} \frac{x^3 + 5x}{2x^3 - x^2 + 4} = \frac{1}{2}$$

Panel 7

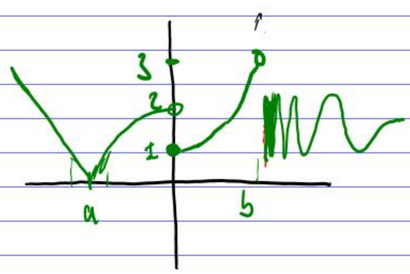
Theorems about Continuous Functions

Panel 8

Name:

Quiz #3

① Consider the graph below.



a) $\lim_{x \to a} f(x)$

b) $\lim_{x \to b^-} f(x)$

c) is f cont. at $x = a$? If not, what type:

d) is f cont. at $x = 0$? If not, what type:

e) is f cont. at $x = 6$? If not, what type:

Panel 9

② If $f(x) = \begin{cases} \dfrac{x^2 - x}{x - 1} & \text{if } x \neq 1 \\ c & \text{if } x = 1 \end{cases}$ choose a value for c so that f is cont. at x=1

③ Find the following limits.

a) $\lim_{x \to \infty} \dfrac{2 - 3x + 2x^2}{3 + 3x}$

b) $\lim_{x \to 0} \dfrac{\sin(x)}{5x}$

Panel 10

Theorems about Continuous Functions

Polynomials are Continuous $\dfrac{x^2 - 7}{x}$

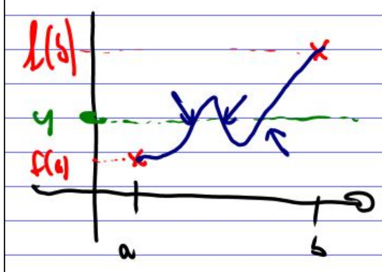
Rational Function are cont. as long as denom is not zero

Compositions are continuous

Intermediate Value Theorem (IVT)

If f is continuous on interval [a,b], and y is any number s.t. $f(a) < y < f(b)$, then there is an x in (a,b) s.t. $f(x) = y$

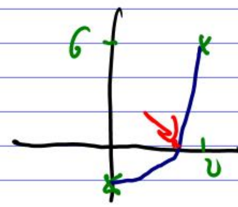
Panel 11



Show that
$f(x) = x^3 - 2$ has a zero
between 0 and 2, i.e.
a: b:

$f(x)=0$ for some x $\quad x^3-2=0 \Rightarrow x^3 = 2,\ x=\sqrt[3]{2}$

$f(a) = f(0) = -2,$
$f(b) = f(2) = 6$



$f(c)=0$ for some c
because $f(a)<0$ and
$f(b)>0$

Of course $x=\sqrt[3]{2}$

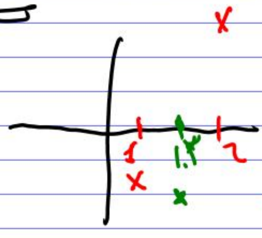
Panel 12

Application: Show that
$$f(x)=4x^3-6x^2+3x-2=0$$
has a solution between 1 and 2

$f(1)=-1$ and $f(2)=\overset{32}{4\cdot 8}-\overset{-24}{6\cdot 4}+\overset{+6}{6}\overset{-2}{-2}=12$, f is cont.

YES: There must be an x s.t. $f(x)=0$

Is $x=1.5$? $f(1.5)=-1.91$



$\Rightarrow$ there is x between 1.5 and 2 s.t. $f(x)=0$

Is $x=1.75$? $\quad f(1.75)=0$ done

$f(1.75)>0 \Rightarrow x\in[1.5,1.75]$

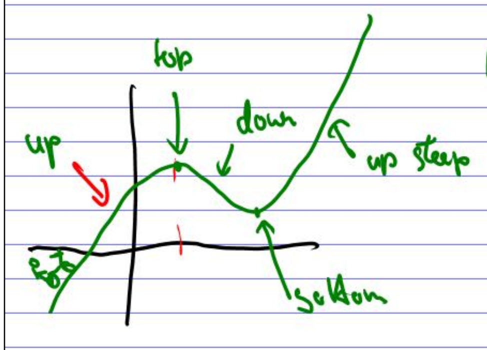
$f(1.75)<0 \Rightarrow x\in[1.75,2]$

Bisection Method

Panel 13

Next: Differentiation

Goal: Investigate the shape of the graph of $f(x)$, whether it goes up or down, or...



Ex: $f(x) = 5x^2$ is distance of a ball dropped from a platform.

What is avg speed after 5 secs? x is time, $f(x)$ is distance

avg. speed is $\frac{\text{difference in distance}}{\text{difference in time}} = \frac{f(5)-f(0)}{5-0} = \frac{5 \cdot 25}{5} = \underline{25}$

Panel 14

$f(t) = 16t^2 + 5$ is distance after t hours of driving a car for t hours

Avg. speed after 2 hours? $f(2) - f(0)$ is dist in $2-0$ hours

$\Rightarrow$ avg. speed $\frac{f(2)-f(0)}{2-0} = \frac{16 \cdot 4 + 5}{2}$

Avg. speed after 3 hours? $\frac{f(3)}{3}$

Avg. speed between 1 and 3 hours: $\frac{f(3)-f(1)}{3-1}$

What is the actual speed at 2 hours??? Next time!