Panel 1

Fundamental Thm of Calc (1) ← Evaluation Thm

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

Ex: $$\int_2^3 3x^2 - \frac{1}{x}\,dx = x^3 - \ln(x)\Big|_2^3 = \left[3^3 - \ln(3)\right] - \left[2^3 - \ln(2)\right]$$
$$= 19 - \ln(3) + \ln(2)$$

Fund. Thm. of Calc (2): If f is cont. on $[a,b]$ and define $F(x) = \int_a^x f(t)\,dt$

Then: F is diff'ble and $\frac{d}{dx} F(x) = f(x)$

$$\cancel{\frac{d}{dx}} \cancel{\int} f(x)\,dx$$

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Panel 2

Ex: Define $g(x) = \int_0^x \sqrt{1+t^2}\,dt$.

Find $g(0)$, $g'(x)$, and $g''(x)$

$$g(0) = \int_0^0 \sqrt{1+t^2}\,dt = 0$$

$$g'(x) = \sqrt{1+x^2} = \cancel{\frac{d}{dx}} \int_0^x \sqrt{1+x^2}\,\cancel{dt} \geq 0 \Rightarrow g \text{ is increasing}$$

$$g''(x) = \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x$$

no easy antideriv!

Note: $$\int \sqrt{1+t^2}\,dt = \int (1+t^2)^{1/2}\,dt = \cancel{\frac{2}{3}(1+t^2)^{3/2} \cdot \frac{1}{2t}}$$

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Panel 3

2nd Fund Thm let's me define new functions

$g(x) = \int_1^x \frac{1}{t}\,dt$ can be used to define $\ln(x)$

$g(1) = 0, \quad g'(x) = \frac{1}{x}$

$g(2) = \int_1^2 \frac{1}{t}\,dt$

$\int_1^2 \frac{1}{t}\,dt \approx \frac{1}{4}\left(f(1.25) + f(1.5) + f(1.75) + f(2)\right)$

$= \frac{1}{4}\left(\frac{1}{1.25} + \frac{1}{1.5} + \frac{1}{1.75} + \frac{1}{2}\right) =$

$= \underline{0.634}$

Note: $g(x) = \ln(x)$. $\Rightarrow \ln(2) \approx 0.634 =$

$= 0.69$

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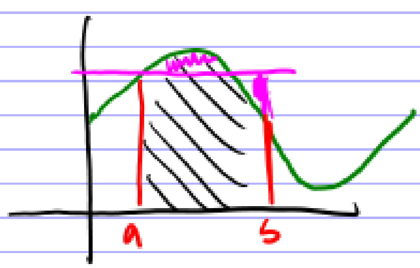
Panel 4

Mean Value Theorem for Integration

If f is cont. on $[a,b]$ then there is a $c \in (a,b)$ s.t.

$\int_a^b f(x)\,dx = f(c)(b-a) \iff \frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$

average of f over $[a,b]$



Recall: MVT (diff.): f diffble on $[a,b]$, $f'(c) = \frac{f(b)-f(a)}{b-a}$

Rolle: f diffble on $[a,b]$, $f(a) = f(b) = 0 \Rightarrow f'(c) = 0$

IVT: f cont. on $[a,b]$, $d \in [f(a), f(b)] \Rightarrow f(c) = d$

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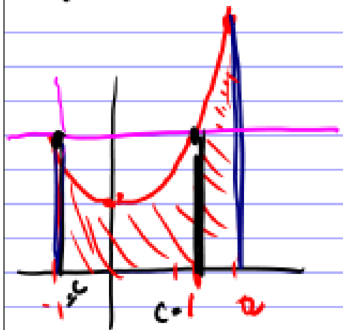
Panel 5

Find the c from the Mean Value Theorem for

$f(x) = 1+x^2$ on $[-1,2]$



$$\int_{-1}^{2} 1+x^2\,dx = x+\frac{1}{3}x^3\Big|_{-1}^{2} =$$

$$\left(2+\frac{8}{3}\right)-\left(-1-\frac{1}{3}\right) =$$

$$(3+3) = 6$$

$$6 = \int_{-1}^{2} 1+x^2\,dx = f(c)(b-a) = (1+c^2)(2-(-1))$$

$$6 = 3(1+c^2) \Rightarrow 2 = 1+c^2 \Rightarrow 1 = c^2 \Rightarrow c = \pm 1$$

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Panel 6

Flash back:

constant

$$\frac{d}{dx}\int_{1}^{5} \frac{\sin(t)}{t}\,dt = 0$$

$$\frac{d}{dx}\underbrace{\int_{1}^{x} \frac{\sin(t)}{t}\,dt}_{f(x)} = \frac{\sin(x)}{x}$$

$$\cancel{\frac{d}{dx}\int_{1}^{x} \frac{f(x)}{x}\,dx}$$

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Panel 7

$$\boxed{\frac{d}{dx}\int_1^x \sin(t)+t^2-e^t\,dt =} \quad \underline{\sin(x)+x^2-e^x}$$

$$\frac{d}{dx}\left(-\cos(t)+\frac{1}{3}t^3-e^t\Big|_1^x = \right)$$

$$\frac{d}{dx}\left[\left[-\cos(x)+\frac{1}{3}x^3-e^x\right]-\left[-\cos(1)+\frac{1}{3}1^3-e^1\right]\right] =$$

$$\underline{\sin(x)+x^2-e^x}$$

$$F(x)=\int_1^x e^{-t^2}\,dt. \quad \to F(1)=0$$

$$F'(x)=e^{-x^2}$$

$$F''(x)=(-2x)e^{-x^2}$$

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Panel 8

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Panel 9

Substitution Rule = "Anti-Chain Rule"

Many antideriv. are tricky because you guess

Ex: $\int 2x(\underbrace{1+x^2}_{u})^2\,dx = \int 2x(1+2x^2+x^4)\,dx =$

$= \int 2x+4x^3+2x^5\,dx =$

$u = 1+x^2$

$= x^2 + x^4 + \frac{2}{6}x^6 + C$

$\frac{du}{dx} = 2x \Leftrightarrow du = 2x\,dx$

$\int 2x(1+x^2)^2\,dx = \int u^2\,du = \frac{1}{3}u^3 + C =$

$= \frac{1}{3}(1+x^2)^3 + C =$

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Panel 10

Substitution Rule: If an integrand is s.t. it includes a function and its derivative then set $u =$ function, compute $\frac{du}{dx} = \_\_\_$ and substitute u and du to get a new problem.

Ex: $\int \underbrace{(2-3x^2)}_{u'}\sqrt{\underbrace{2x-x^3}_{u}}\,dx = \int (u)^{1/2}\,du =$

$u = 2x-x^3$

$= \frac{2}{3}u^{3/2} + C =$

$\frac{du}{dx} = (2-3x^2)\,dx$

$= \frac{2}{3}(2x-x^3)^{3/2} + C$

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Panel 11

$$\int x\sqrt{1+x}\,dx = \int x\sqrt{u}\,du = \int (u-1)\sqrt{u}\,du$$

$u = 1+x \Rightarrow x = u-1$

$\frac{du}{dx} = 1, \quad du = dx$

$$= \int u^{3/2} - u^{1/2}\,du =$$

$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{5}(1+x)^{5/2} - \frac{2}{3}(1+x)^{3/2} + C$$

Check $\frac{d}{dx}(*) = (x*)$

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