

Panel 1

Review: *indef. integral, antiderivative*

$$\int f(x) dx = \text{function} + C$$

*def. integral*  

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$
*limit of Riemann Sum*  
*area under a curve*  
*Function of calculus*  
*number*  
*F is antideriv of f*

Panel 2

$$\int 4x^5 - 3x^4 + \sqrt{x^2} - \frac{7}{x^3} + 5e^x + \frac{1}{x} + 5 dx$$

$$\frac{4}{6} x^6 - 3 \frac{1}{5} x^5 + \frac{2}{3} x^{3/2} - 7 \frac{1}{(-2)} x^{-2} + 5e^x + \ln(x) + 5x + C$$

$$\int_{\pi}^0 5x^4 - \cos(x) = x^5 - \sin(x) \Big|_{\pi}^0 =$$

$$\left( 0^5 - \sin(0) + C \right) - \left( \pi^5 - \sin(\pi) + C \right) = -\pi^5$$

Find  $f$  s.t.  $f'(x) = 7x^2 - \sec^2(x)$  and  $f(0) = 10$

$$f(x) = \int 7x^2 - \sec^2(x) dx = \frac{7}{3} x^3 - \tan(x) + C$$

$$f(0) = \frac{7}{3} 0^3 - \tan(0) + C = 10$$

Panel 3

Last Quiz

Name: \_\_\_\_\_

① Find the following antiderivatives:

a)  $\int 3x^2 + \sqrt[3]{x^7} - \frac{3}{x^5} dx$

b)  $\int 5e^x - \frac{1}{x} + \sin(x) dx$

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Panel 4

② Evaluate the following definite integrals:

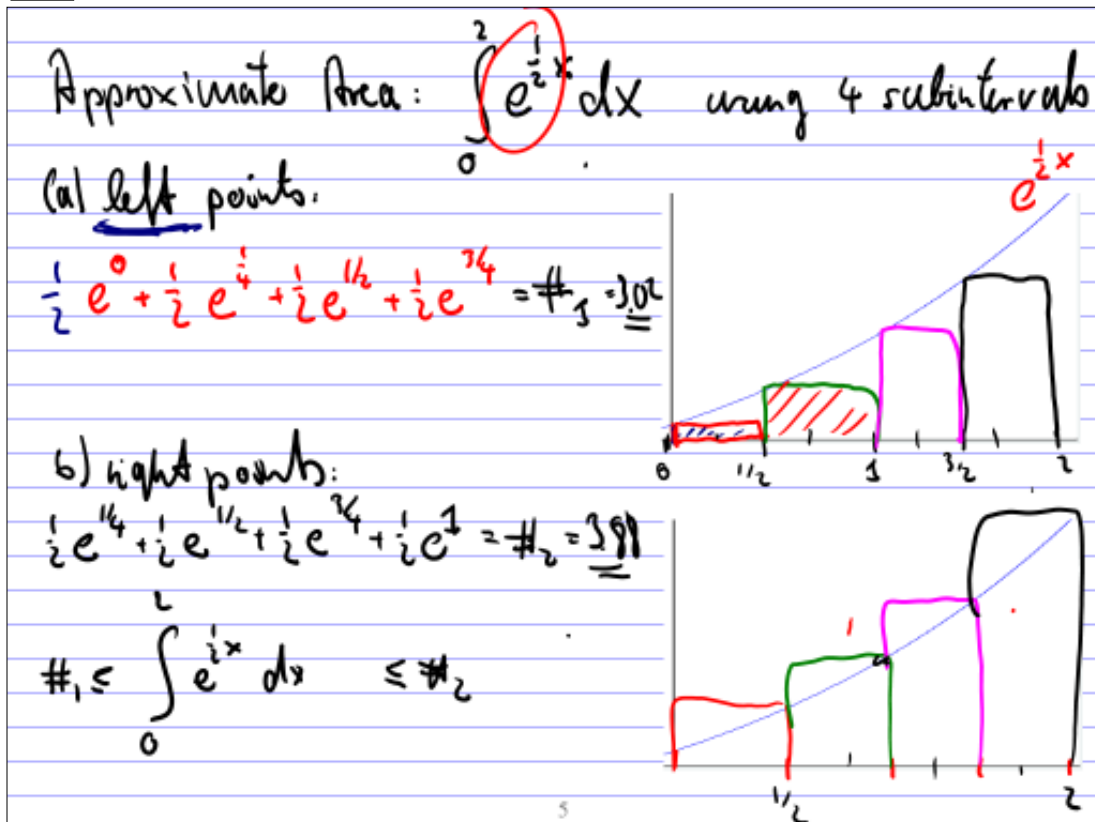
a)  $\int_0^2 6x^2 - 4x dx$

b)  $\int_1^e 5 + \frac{1}{x} dx$

③ If  $v(t) = 2t$  is the speed of a particle, find the distance function  $s(t)$ , where  $s(0) = 10$ 

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Panel 5



Panel 6

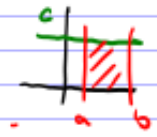
$$\int_0^2 e^{\frac{1}{2}x} dx = 2 e^{\frac{1}{2}x}$$

$$\left( \frac{d}{dx} \right) 2e^{\frac{1}{2}x} \cdot \frac{1}{2}$$

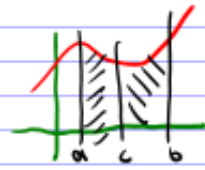
$$= 2e^{\frac{1}{2}x} \Big|_0^2 = 2e^1 - 2e^0 = \underline{\underline{3.4365}}$$

Panel 7

Properties of the Integral

(1)  $\int_a^b c \, dx = (b-a)c$  

(2)  $\int_a^b f(x) + g(x) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx$

(3)  $\int_a^b c \cdot f(x) \, dx = c \int_a^b f(x) \, dx$  

(4)  $\int_a^c f(x) \, dx + \int_c^b f(x) \, dx = \int_a^b f(x) \, dx$

(5) If  $f(x) \geq g(x)$  then  $\int_a^b f(x) \, dx \geq \int_a^b g(x) \, dx$

(6) If  $m \leq f(x) \leq M$  then  $\int_a^b m \, dx \leq \int_a^b f(x) \, dx \leq \int_a^b M \, dx$   
 $(b-a)m \leq \int_a^b f(x) \, dx \leq (b-a)M$

If  $f$  is cont. on  $[a,b]$  then  $f$  is integrable!

Panel 8

Ex: Find estimates for  $\int_0^1 e^{-x^2} \, dx \leq 1(1-0)$

$\rightarrow \frac{1}{e} \int_0^1 e^{-x^2} \, dx \leq 1$

Try:  $\int e^{-x^2} \, dx = \frac{1}{-2x} e^{-x^2}$  ~~has no antiderivative~~

$\frac{d}{dx} e^{-x^2} = e^{-x^2} (-2x) < 0 \quad \forall x > 0$   
 $\rightarrow$  decreasing on  $[0,1] \Rightarrow$  max at  $x=0 \Rightarrow 1$   
 min at  $x=1 \Rightarrow \frac{1}{e}$

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Panel 9

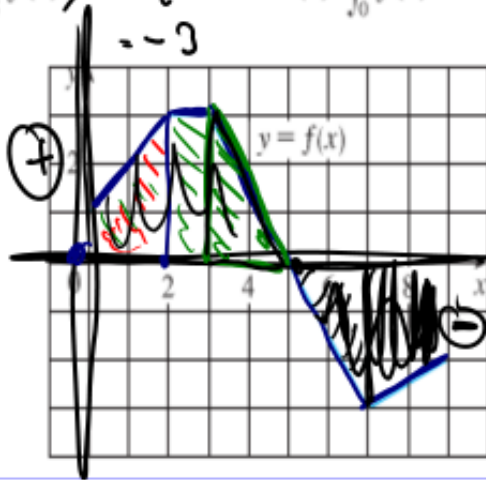
29. The graph of  $f$  is shown. Evaluate each integral by interpreting it in terms of areas.

(a)  $\int_0^2 f(x) dx = 4$

(b)  $\int_0^5 f(x) dx = 4 + 3 + \frac{1}{2} \cdot 2 \cdot 3 = 10$

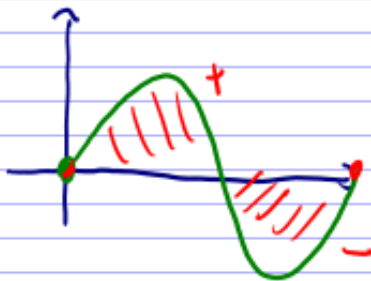
(c)  $\int_5^7 f(x) dx = -\frac{1}{2} \cdot 2 \cdot 3 = -3$

(d)  $\int_0^9 f(x) dx = \text{above x-axis} - \text{below ground}$



Panel 10

$$\int_0^{2\pi} \sin(x) dx = -\cos(x) \Big|_0^{2\pi} = (-\cos(2\pi)) - (-\cos(0)) = -1 - (-1) = 0$$



+ area and "-area" cancel out.

$\int_a^b f(x) dx$  is area under curve if  $f \geq 0$ .

Otherwise, it is total signed area!

Panel 11

Fundamental Thm of Calc (1)

Ex:  $\int_1^2 3x^2 - \frac{1}{x} dx$

~~Friday~~

Fund. Thm. of Calc (2): If  $f$  is cont. on  $[a,b]$   
and define  $F(x) =$

Then: