Panel 1

Last Time

Antideriv. of x^2

$\to F(x) = \frac{1}{3}x^3 + c$

Applications of Antiderivatives

Know: $a(t) = -10\ m/sec^2 \to v(t) = \int -10\,dt = -10t + c$

$\to s(t) = \int v(t)\,dt = \int -10t + c\,dt = -5t^2 + ct + d$

$$\int f(x)\,dx = F(x) + c \qquad \int x^2\,dx = \frac{1}{3}x^3 + c$$

integral of ___ dx

indefinite integral

antideriv.

Area under Curve: Review

1

Panel 2

$$\int x^3 + \sqrt{x} - \frac{7}{x^2} + \pi^2\,dx =$$

$x^{1/2}$ $\quad -7x^{-2}$

$$\frac{1}{4}x^4 + \frac{1}{3/2}x^{3/2} + 7\,x^{-1} + \pi^2 \cdot x + C$$

$$p \neq -1 \quad \int x^p\,dx = \frac{1}{p+1}x^{p+1} + c$$

$+9x^{-1/3}$ $\quad -5x^{-1}$

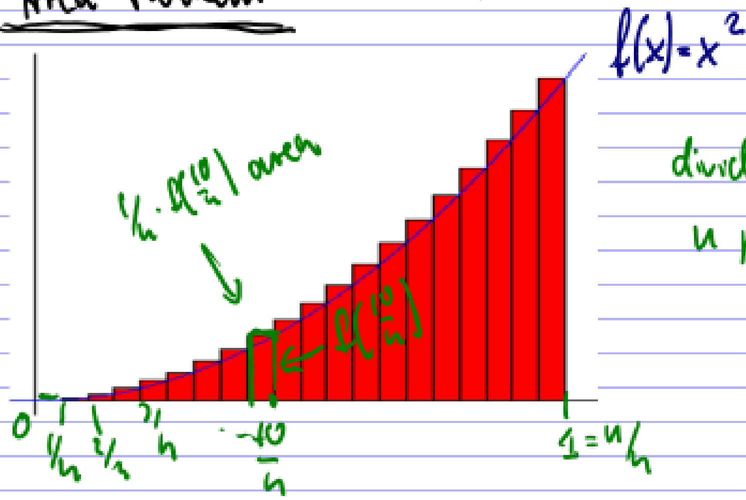
Ex) $$\int 7x^5 - 5e^x + \frac{9}{\sqrt[3]{x}} - \frac{5}{x} + 3\cos(x) + 1\,dx =$$

$$7\tfrac{1}{6}x^6 - 5e^x + 9\tfrac{3}{2}x^{2/3} - 5\ln|x| + 3\sin(x) + 1 \cdot x + C$$

2

Panel 3

Area Problem



divide [0,1] into n parts

Red Area: $\frac{1}{n} f\left(\frac{1}{n}\right) + \frac{1}{n} f\left(\frac{2}{n}\right) + \frac{1}{n} f\left(\frac{3}{n}\right) + \frac{1}{n} f\left(\frac{4}{n}\right) + \ldots + \frac{1}{n} f\left(\frac{n}{n}\right) =$

$$= \frac{1}{n}\left(\frac{1}{n^2} + \frac{4}{n^2} + \frac{9}{n^2} + \frac{16}{n^2} + \ldots \frac{n^2}{n^2}\right) = \frac{1}{n} \cdot \frac{1}{n^2}\left(1+4+9+16+\ldots+n^2\right)$$

3

Panel 4

Sock Sum

$$\begin{aligned}
1+2+3+4+\ldots+98+99+100 &= S\\
100+99+\ldots+3+2+1 &= S\\
\hline
101+101+101+\ldots\ 101+101+101 &= 2S
\end{aligned}$$

$$100 \cdot 101 = 2S \Rightarrow S = \frac{100 \cdot 101}{2} = 50 \cdot 101 = \underline{5050}$$

$$\begin{aligned}
1+2+3+\ldots+(n-1)+n &= S\\
n+(n-1)+\ldots\ +2+1 &= S\\
\hline
(n+1)+(n+1)\ \ldots\ (n+1)+(n+1) &= 2S
\end{aligned}$$

$$\Rightarrow 2S = n(n+1)$$

$$\underline{S = \frac{n(n+1)}{2}}$$

4

Panel 5

$$1+2+3+\dots+(n-1)+n = \frac{n(n+1)}{2}$$

$$1+4+9+16+\dots+(n-1)^2+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1+2^3+3^3+4^3+\dots+(n-1)^3+n^3 = \underline{\qquad\qquad}$$

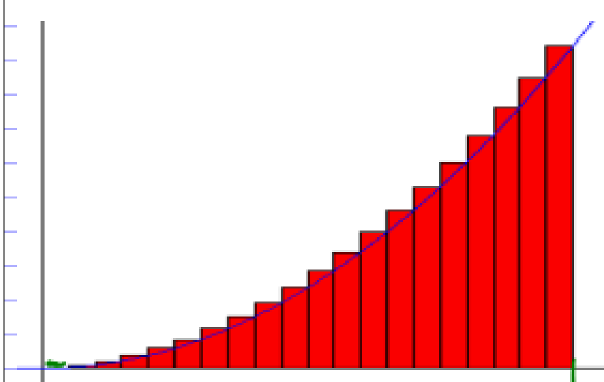
5

Panel 6

Red Area: $\frac{1}{n} f\left(\frac{1}{n}\right)+\frac{1}{n} f\left(\frac{2}{n}\right)+\frac{1}{n} f\left(\frac{3}{n}\right)+\frac{1}{n} f\left(\frac{4}{n}\right)+\dots+\frac{1}{n} f\left(\frac{n}{n}\right)=$

$$=\underbrace{\frac{1}{n}\left(\frac{1}{n^2}+\frac{4}{n^2}+\frac{9}{n^2}+\frac{16}{n^2}+\dots+\frac{n^2}{n^2}\right)}_{(*)} = \frac{1}{n}\cdot\frac{1}{n^2}\left(1+4+9+16+\dots+n^2\right)$$

$$=\frac{1}{n^3}\,\frac{n(n+1)(2n+1)}{6}$$



Thus: area under the curve is:

$$A=\lim_{n\to\infty}(*)=$$

$$=\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}=\frac{2}{6}=1/3$$

6

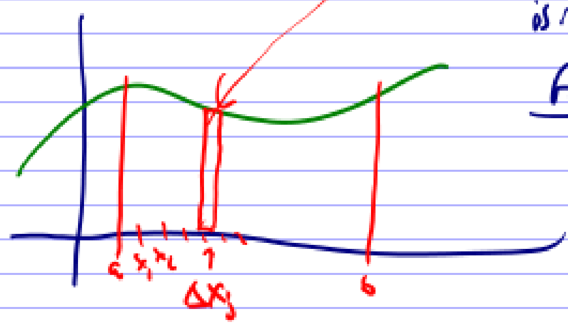
Panel 7

Definition of Definite Integral

f defined on $[a,b]$. Divide $[a,b]$ into n subintervals of length Δx_j. Then define

$$\lim_{\Delta x \to 0} f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + \dots + f(x_n)\Delta x_n = \int_a^b f(x)\,dx$$

is: definite integral of $f(x)$ from a to b



7

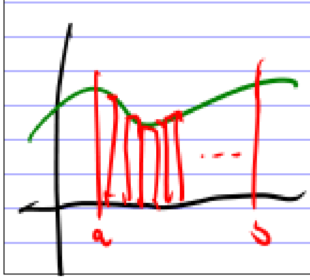
Panel 8

"Simple"

Call $f(x_1)\Delta x_1 + \dots + f(x_n)\Delta x_n$ a Riemann Sum

Then: $\displaystyle\int_a^b f(x)\,dx$ = limit of Riemann Sums

Geometric interpretation: $\displaystyle\int_a^b f(x)\,dx$ = area under $f(x)$ from a to b (if $f(x) > 0$)



8

Panel 9

Simpler Definition of definite Integral

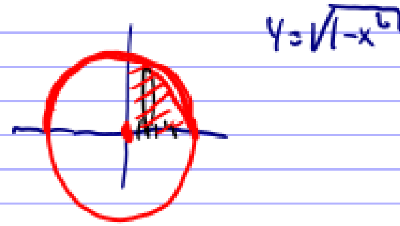
~~done~~

9

Panel 10

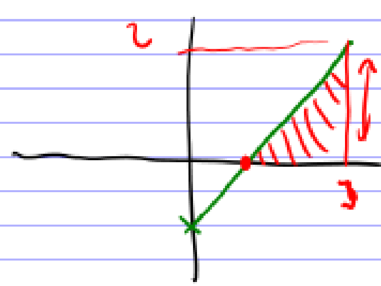
Ex: Find, any way possible:

a) $\int_0^1 \sqrt{1-x^2}\,dx = \pi/4$



b) $\int_1^3 x-1\,dx = \frac{1}{2} b \cdot h =$

$= \frac{1}{2} \cdot 3 \cdot 2 = \underline{3}$



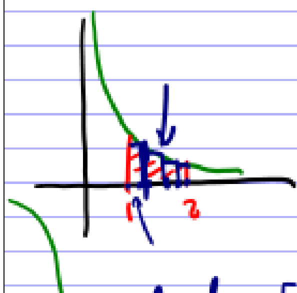
10

Panel 11

Approximate $\int_1^2 \frac{1}{x}\,dx \approx \frac{1}{4} f\left(\frac{5}{4}\right)+\frac{1}{4} f\left(\frac{6}{4}\right)+\frac{1}{4} f\left(\frac{7}{4}\right)+\frac{1}{4} f\left(\frac{8}{4}\right)$

$= \frac{1}{4}\left(\frac{4}{5}+\frac{4}{6}+\frac{4}{7}+\frac{4}{8}\right) = \underline{0.634}$

true area is a little larger than 0.634



divide [1,2] into 4 parts:

$x_1=\frac{5}{4},\ x_2=\frac{6}{4},\ x_3=\frac{7}{4},\ x_4=\frac{8}{4}$

11

Panel 12

Big-Deal Theorem: Fundamental Thm of Calculus

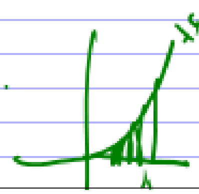
If f is integrable on $[a,b]$ (i.e. limit of Riemann Sums exists)

then $\int_a^b f(x)\,dx = F(x)\Big|_{x=a}^{x=b} = F(b)-F(a)$

where F is an antiderivative of $f(x)$

Ex: $\int_0^1 x^2\,dx \left(=\frac{1}{3}\right) = \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3}(1)^3-\frac{1}{3}(0)^3 = 1/3$

$\int_0^1 x^4\,dx = \frac{1}{5}x^5\Big|_0^1 = \frac{1}{5}-0 = \frac{1}{5}$

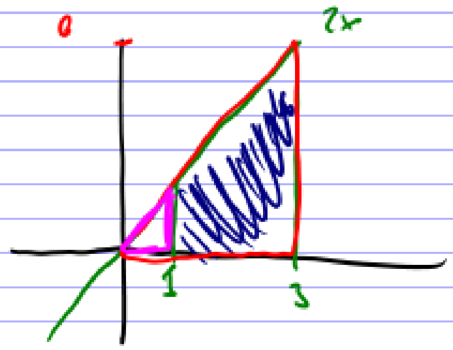


12

Panel 13

Compute $\int_1^3 2x\,dx$ geometrically and alg.

$$\int_1^3 2x\,dx = 2\tfrac{1}{2}x^2\Big|_1^3 = x^2\Big|_1^3 = 9-1=8$$



$\Delta = \frac{1}{2}\cdot 3\cdot 6 = 9$

$\Delta = \frac{1}{2}\cdot 1\cdot 2 = 1$

Area $= 9-1 = \underline{8}$

13

Panel 14

Quiz on Wed (our last)

on antideriv. $\int f(x)\,dx$ (= function)

and def. integrals $\int_a^b f(x)\,dx$ (= number)

14