Panel 1

Last Time

Exp. growth / decay: $y = ae^{kx}$ / $P = P_0 e^{kt}$ $k>0$: growth, $k<0$: decay

$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ Domain: $[-\pi/2, \pi/2]$

$\frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$ Domain: $[0, \pi]$

$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$ Domain: $(-\pi/2, \pi/2)$

implicit differentiation: see before

$\frac{d}{dx} e^x = e^x$ $\frac{d}{dx} \ln|x| = 1/x$

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Panel 2

Quiz 12

Name: __________

① A cell culture, prepared at noon with 100 cells, grew to 400 cells 2 hours later. How many cells will the culture contain at 6pm, assuming exponential growth?

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Panel 3

② Find the derivatives of the following functions.

a) $f(x) = \ln(x) \cdot e^{3x^2}$

b) $g(x) = \sin^{-1}(1-x) + \cos^{-1}(1-x)$

c) $h(x) = \tan^{-1}(e^{x^2})$

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Panel 4

Draw graph of $\tan(x)$ and $\tan^{-1}(x)$

Domain: $\tan(x) = \frac{\sin(x)}{\cos(x)}$,

$x \in (-\frac{\pi}{2}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2}) \dots$

Asympt.: $\lim_{x \to \pm\pi/2} \tan(x) = \pm\infty$

critical $f'(x) = \sec^2(x) > 0$

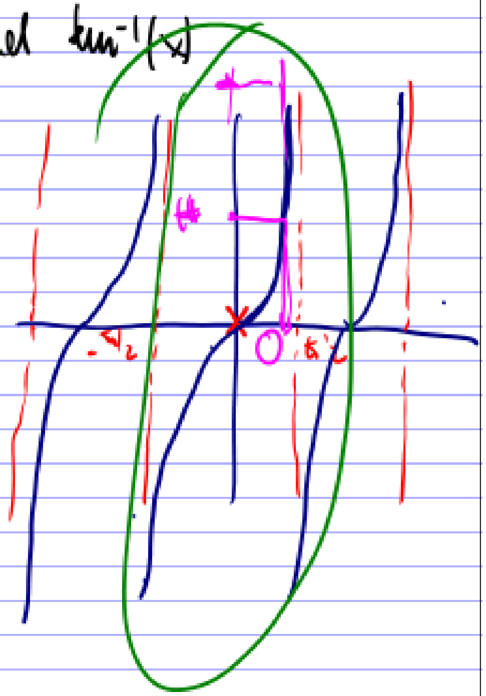
$\rightarrow f$ is increasing

infl.: $f''(x) = 2\sec^2(x)\tan(x) = 0$

$x = 0$

f''

f



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Panel 5

$f(x) = \tan^{-1}(x)$

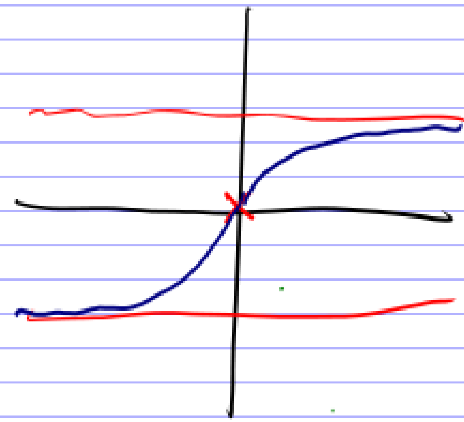
<u>Domain:</u> $\mathbb{R}$

$\lim_{x \to \pm\infty} \tan^{-1}(x) = \pm\frac{\pi}{2}$

$f'(x) = \frac{1}{1+x^2} = (1+x^2)^{-1} = 0$ none

$f''(x) = -(1+x^2)^{-2} \cdot 2x =$

$-\frac{2x}{(1+x^2)^2} = 0 \qquad x = 0$



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Panel 6

Consider $f(x) = \arctan(e^x)$. Find domain and f'.

$= \tan^{-1}(e^x)$

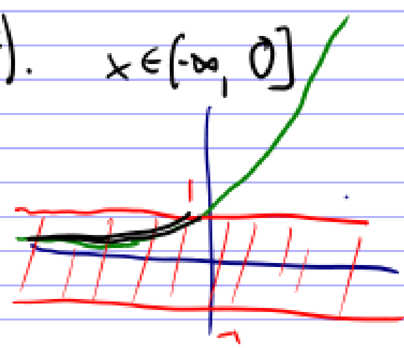
Domain: $\mathbb{R}$

$f'(x) = \frac{1}{1+(e^x)^2} \cdot e^x = \frac{e^x}{1+e^{2x}}$

Same for $g(x) = \arcsin(e^x)$. $\quad x \in (-\infty, 0]$

$= \sin^{-1}(e^x)$

$\sin^{-1}(x)$ has domain $(-1,1)$



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Panel 7

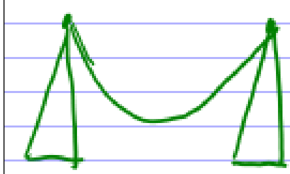
Hyperbolic Trig Functions

Def: $\sinh(x) = \frac{1}{2}(e^{x} - e^{-x}) \Rightarrow \frac{d}{dx}\sinh(x) = \frac{1}{2}(e^{x} + e^{-x}) = \cosh(x)$

$\cosh(x) = \frac{1}{2}(e^{x} + e^{-x}) \Rightarrow \frac{d}{dx}\cosh(x) = \frac{1}{2}(e^{x} - e^{-x}) = \sinh(x)$

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ HW

Claim to fame: If a wire hangs between two poles, it is shaped like



$$y = c + a\cosh\left(\frac{x}{a}\right)$$

is called catenary (chain)

Panel 8

Ex: Find the derivatives of the hyperbolic trig functions.

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Panel 9

Inverse Hyperbolic Trig Functions

very important

$f(x) = \sinh^{-1}(x) = \ln(x+\sqrt{x^2+1})$

$\frac{d}{dx} \sinh^{-1}(x)$

$f(x) = \cosh^{-1}(x) = \ln(x+\sqrt{x^2-1})$

$\frac{d}{dx} \cosh^{-1}(x)$

$f(x) = \tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

$\frac{d}{dx} \tanh^{-1}(x) =$

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Panel 10

l'Hospital's Rule

Nifty trick to find difficult limits

Thm: Suppose f, g are diff'ble at $x=c$ and

$f(c) = g(c) = 0$

$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$

Also, if $\lim_{x\to\infty} f(x) = \pm\infty$ and $\lim_{x\to\infty} g(x) = \pm\infty$ then

$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$

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Panel 11

Ex: l'Hospital's Rule is perfect for tricky limits

$$\lim_{x\to 0} \frac{x^2-9}{x-3} = \frac{-9}{-3} = 3$$

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{2x}{1} = 6$$

$$\lim_{x\to 0} \frac{\sin(x)}{x} = \lim_{x\to 0} \frac{\cos(x)}{1} = 1$$

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x^3} = \lim_{x\to 0} \frac{2xe^{x^2}}{3x^2} = \lim_{x\to 0} \frac{2}{3}\frac{e^{x^2}}{x} = \pm\infty \text{ or undef.}$$

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Panel 12

$$\lim_{x\to 1} \frac{\ln(x)}{x-1} = \lim_{x\to 1} \frac{1/x}{1} = 1$$

$$\lim_{x\to\infty} \frac{e^x}{x^2}$$

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[3]{x}}$$

$$\lim_{x\to 0^+} x\ln(x)$$

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