Panel 1

Last Time.

(Q4 on Wed)

$f(x) = e^x \Rightarrow f'(x) = e^x$

$f(x) = a^x = e^{\ln(a^x)} = e^{x\ln(a)} \Rightarrow f'(x) = e^{x\ln(a)} \cdot \ln(a) = a^x \ln(a)$

$f(x) = \ln|x| \Rightarrow f'(x) = \frac{1}{x}$

$f(x) = \log_5(x) = \frac{\ln(x)}{\ln(5)} \Rightarrow f'(x) = \frac{1}{\ln(5)} \cdot \frac{1}{x}$

$\log_5(x) = y \Leftrightarrow 5^y = x \quad | \ln$

$y\ln(5) = \ln(x) \Rightarrow y = \frac{\ln(x)}{\ln(5)}$

$y = \frac{x^5\sqrt{x-1}}{(x^2-2)^2}$

$\ln(y) = 5\ln(x) + \frac{1}{2}\ln(x-1) - 2\ln(x^2-2)$

$\frac{1}{y}y' = \frac{5}{x} + \frac{1}{2(x-1)} - \frac{2\cdot 2x}{x^2-2}$

Exp. growth / decay: $\boxed{y = ae^{kx}}$

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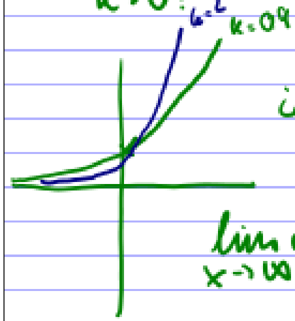
Panel 2

Exp Growth / Decay

$y = ae^{kx}$

Ex: $y = ae^{-2x} = a\left(\frac{1}{e}\right)^{\dots}$

$k > 0$:

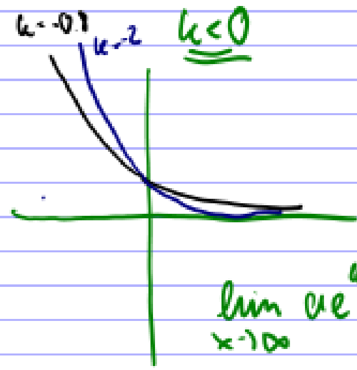


increasing

$\lim_{x\to\infty} ae^{kx} = \infty$

Exp. Growth

$k < 0$



$\lim_{x\to\infty} ae^{kx} = 0$

Exp. Decay

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Panel 3

EXAMPLE 1 Use the fact that the world population was 2560 million in 1950 and 3040 million in 1960 to model the population of the world in the second half of the 20th century. (Assume that the growth rate is proportional to the population size.)

What is the population in 2020? $\Rightarrow P(70) = 2560e^{0.0172\cdot 70}$

$P(t)$ is pop size at time t, where $t=0 \Leftrightarrow 1950$

$\Rightarrow$ Model $P(t) = P_0 e^{kt}$, P_0 is $P(t=0)$ initial pop

$\Rightarrow P(0) = 2560 = P_0 e^{k0} = P_0 \Rightarrow P_0 = 2560$

$\Rightarrow P(10) = 2560e^{10k} = 3040$ $\quad P(t) = 2560e^{0.0172t}$

$\Rightarrow e^{k10} = \frac{3040}{2560}$ / ln $\Rightarrow 10k = \ln\left(\frac{3040}{2560}\right) \Rightarrow k = \frac{1}{10}\ln(\quad) = 0.0172$

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Panel 4

EXAMPLE 2 The half-life of radium-226 ($^{226}_{88}Ra$) is 1590 years.

(a) A sample of radium-226 has a mass of 100 mg. Find a formula for the mass of $^{226}_{88}Ra$ that remains after t years.

(b) Find the mass after 1000 years correct to the nearest milligram.

(c) When will the mass be reduced to 30 mg?

Model. $A(t) = A_0 e^{kt}$, $k<0$

$A(0) = A_0$

$\Rightarrow A(1590) = \frac{1}{2}\cancel{A_0} = \cancel{A_0} e^{k\cdot 1590}$ / ln

$\ln(\frac{1}{2}) = 1590k$

$\frac{1}{1590}\ln(\frac{1}{2}) = k \Rightarrow \boxed{k = -\frac{\ln(2)}{1590}}$

$\ln(\frac{1}{2}) = \cancel{\ln(1)}^{0} - \ln(2) = -\ln(2)$

$\log_b(1) = y : b^y = 1, y = 0$

a) $A(t) = 100e^{-\frac{\ln(2)}{1590}\cdot t}$

b) $A(1000) = 100e^{-\frac{\ln(2)}{1590}\cdot t}$

c) Want $A(t) = 30$

$100e^{-\frac{\ln(2)}{1590}t} = 30$

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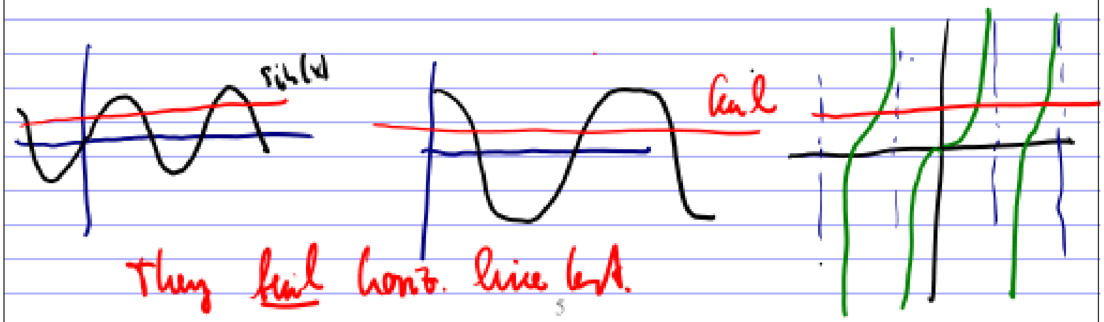
Panel 5

Inverse Trig Functions

Our study of inverse functions led us to the discovery of a new function: $\ln(x)$, $\log_b(x)$

Trig again: Inverse of sin, cos, tan

Problem: none of them have inverse!!

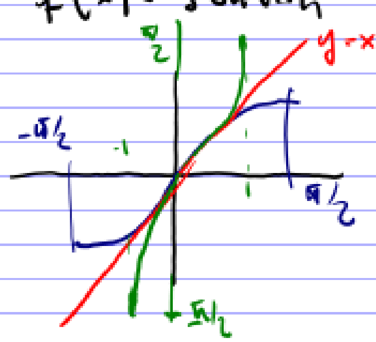


They fail horiz. line test.

Panel 6

Inverse sin Function

Def: $f(x) = \sin(x)$, $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$



Define $f^{-1}(x) = \sin^{-1}(x)$

via

$y = \sin^{-1}(x) \Leftrightarrow \sin(y) = x$

$x \in [-1,1]$, $y \in [-\pi/2, \pi/2]$

domain

Ex: $\sin^{-1}(0) = y \Rightarrow \sin(y) = 0 \Rightarrow y = 0$

$\sin^{-1}(\frac{1}{2}) = y \Rightarrow \sin(y) = 1/2 \Rightarrow y = \pi/6$, $\sin(\pi/6) =$

Sometimes people write $\sin^{-1}(x) = \arcsin(x)$

Panel 7

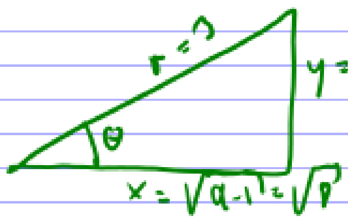
We can do more sophisticated calculations:

Ex: $\tan(\sin^{-1}(1/3))$

$\sin^{-1}\left(\frac{1}{3}\right) = \theta \quad \Big| \sin$

$\cancel{\sin}(\cancel{\sin^{-1}}(\frac{1}{3})) = \sin(\theta)$

$\frac{y}{r} = \frac{1}{3} = \sin(\theta)$



$\tan(\sin^{-1}(\frac{1}{3})) = \tan(\theta) = \frac{1}{\sqrt{8}}$

Recall: $\sin(\theta) = \frac{y}{r}$

$\cos(\theta) = \frac{x}{r}$

$\tan(\theta) = \frac{y}{x}$

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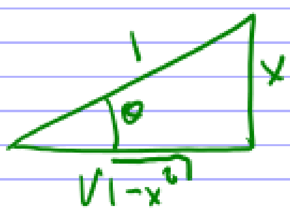
Panel 8

Derivative of $\sin^{-1}$

$y = \sin^{-1}(x) \iff \sin(y) = x \quad \Big| \frac{d}{dx}$

$\cos(y) \cdot y' = 1$

$y' = 1/\cos(y) = \frac{1}{\cos(\sin^{-1}(x))}$



$\sin^{-1}(x) = \theta$

$\frac{x}{1} = x = \sin(\theta)$

$\cos(\sin^{-1}(x)) = \cos(\theta) = \frac{\sqrt{1-x^2}}{1}$

$$\boxed{\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}}$$

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Panel 9

$f(x) = \sin^{-1}(x^2-1)$. Find f'

$f'(x) = \frac{1}{\sqrt{1-(x^2-1)^2}} \cdot 2x = \frac{2x}{\sqrt{2x^2-x^4}}$

$g(x) = x \cdot \cos^{-1}(4x)$. Find g'

$g'(x) = \cos^{-1}(4x) + x \frac{-1}{\sqrt{1-(4x)^2}} \cdot 4 =$

$= \cos^{-1}(4x) - \frac{4x}{\sqrt{1-16x^2}}$

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Panel 10

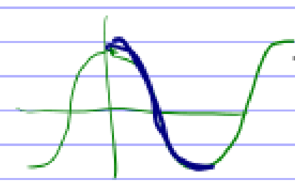
Summary

$\sin(x)$, $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$

$\cos(x)$, $x \in [0, \pi]$

$\frac{d}{dx} \cos^{-1}(x) = -1/\sqrt{1-x^2}$ HW



$\tan(x)$, $x \in [-\pi/2, \pi/2]$

$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$ HW

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