Panel 1

Last Time.

$f(x) = e^x$ and $f(x) = a^x$

$\ln(x) = y \iff e^y = x$ $\qquad \log_3(27) = x = 3$

$\log_b(x) = y \iff b^y = x$ $\qquad 3^x = 27$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

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Panel 2

Find the derivatives of:

$f(x) = \ln(2 + \sin(x)) \Rightarrow f'(x) = \frac{1}{2+\sin(x)} \cdot \cos(x)$

$g(x) = \ln(x^3+1) \qquad f'(x) = \frac{1}{x^3+1} \cdot 3x^2$

$h(x) = \ln(\cos(x)) \qquad f'(x) = 1/\cos(x) \cdot (-\sin(x)) = -\tan(x)$

$k(x) = \sqrt{\ln(1-x)} \qquad f'(x) = \frac{1}{2}(\ln(1-x))^{-1/2} \cdot \frac{1}{1-x}(-1)$

$i(x) = \ln\left(\frac{x+1}{\sqrt{x-2}}\right) = \ln(x+1) - \frac{1}{2}\ln(x-2)$

$f'(x) = \frac{1}{x+1} \cdot 1 - \frac{1}{2} \cdot \frac{1}{x-2} \cdot 1$

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Panel 3

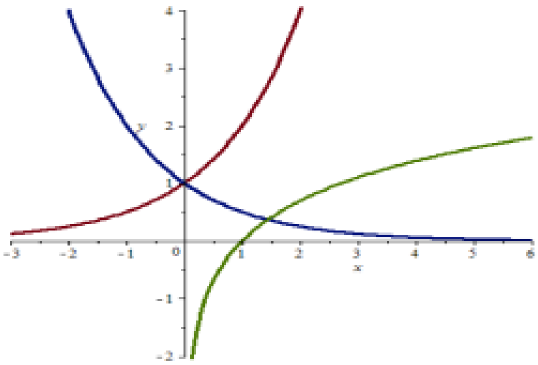
Quiz 11

Name: ______

① Which graph belongs to what function?

(a) $y = 2^x$ (b) $= \left(\frac{1}{2}\right)^x$

(c) $y = \ln(x)$



② Evaluate

(a) $\log_2(32)$

(b) $\log_3(135) - \log_3(5)$

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Panel 4

③ Find the following derivatives

a) $f(x) = \ln(2x)$

b) $f(x) = \sqrt{\ln(\sin(x))}$

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Panel 5

Find $\frac{d}{dx}\log_5(x) = \frac{d}{dx}\frac{1}{\ln(5)}\cdot\ln(x) = \frac{1}{\ln(5)}\cdot\frac{1}{x}$

$\log_5(x) = y \Rightarrow 5^y = x \quad |\ln$

$\ln(5^y) = \ln(x)$

$y\ln(5) = \ln(x)$

$y = \frac{\ln(x)}{\ln(5)}$

Change of Base formula: $\log_5(x) = \frac{\ln(x)}{\ln(5)}$

$\frac{d}{dx}\log_5(x) = \frac{1}{\ln(5)}\cdot\frac{1}{x}$

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Panel 6

Differentiate $y = \frac{x^{3/4}\cdot\sqrt{x^2+1}}{(3x+2)^5}$ $|\ln(\ )$

$\ln(y) = \ln\left(\frac{x^{3/4}\cdot\sqrt{x^2+1}}{(3x+2)^5}\right) =$

$= \ln(x^{3/4}) + \ln(\sqrt{x^2+1}) - \ln((3x+2)^5) =$

$= \frac{3}{4}\ln(x) + \frac{1}{2}\ln(x^2+1) - 5\ln(3x+2) \quad \Big|\frac{d}{dx}$

$\frac{d}{dx}\ln(y) = \frac{3}{4}\cdot\frac{1}{x} + \frac{1}{2}\frac{1}{x^2+1}\cdot 2x - 5\frac{1}{3x+2}\cdot 3$

$\frac{1}{y}\cdot y'$

$\Rightarrow y' = \left(\frac{3}{4}\frac{1}{x} + \frac{1}{2(x^2+1)} - \frac{15}{3x+2}\right)\cdot\frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$

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Panel 7

Logarithmic Differentiation

$$y = \frac{(x-7)^2 \cdot \cos^3(x)}{\tan^5(x)\sqrt{x}}$$

$$\ln(y) = 2\ln(x-7) + 3\ln(\cos(x)) - 5\ln(\tan(x)) - \frac{1}{2}\ln(x) \quad \Big| \frac{d}{dx}$$

$$\frac{1}{y}\, y' = \frac{2}{x-7} - 3\frac{\sin(x)}{\cos(x)} - 5\frac{1}{\tan(x)}\sec^2(x) - \frac{1}{2x}$$

$$y' = (\qquad\qquad\qquad\qquad) \cdot (\qquad)$$

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Panel 8

Derivative of Exp. Function:

Recall: $\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'(f^{-1}(x))}$

$f^{-1}(x) = e^x \Rightarrow f(x) = \ln(x) \Rightarrow f'(\_) = \frac{1}{\_}$

$$\frac{d}{dx} e^x = \frac{d}{dx} f^{-1}(x) = \frac{1}{1/e^x} = e^x$$

Thm: $\frac{d}{dx}\ln(x) = \frac{1}{x}$ and $\frac{d}{dx} e^x = e^x$

"think: $\frac{d}{dx}$ ln is 1 over, $\frac{d}{dx}$ e is itself"

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Panel 9

For thm. Say know $\frac{d}{dx} e^x = e^x$

$\Rightarrow \frac{d}{dx} \ln(x)$

$f^{-1}(x) = \ln(x) \quad \Rightarrow f(x) = e^x$

$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))} = 1/e^{\ln(x)} = \frac{1}{x}$

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Panel 10

Find $\frac{d}{dx} e^{\underline{3x}} = e^{3x} \cdot 3 = \underline{\underline{3e^{3x}}}$

Find $\frac{d}{dx} a^x = \frac{d}{dx} e^{x \cdot \ln(a)} = e^{x \ln(a)} \cdot \ln(a) = a^x \ln(a)$

$a^x = e^{\ln(a^x)} = e^{x \ln(a)}$

Thm: $\frac{d}{dx} \log_a(x) = \frac{1}{\ln(a)} \cdot \frac{1}{x}$

$\frac{d}{dx} a^x = \ln(a) \cdot a^x$

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Panel 11

If $f(x) = e^{-4x} \cdot \sin(5x)$, find $f'(x)$.

$f'(x) = (-4)e^{-4x} \cdot \sin(5x) + e^{-4x} \cos(5x) \cdot 5 = e^{-4x}\left(-4\sin(5x) + 5\cos(5x)\right)$

If $f(x) = e^{x\sin(x)}$, find $f'(x)$

$f'(x) = e^{x\sin(x)} \cdot (\sin(x) + x\cos(x))$

Differentiate $f(x) = x^x = x^x(\ln(x)+1)$

$x^x = e^{\ln(x^x)} = e^{x \cdot \ln(x)}$

$\Rightarrow \frac{d}{dx} e^{x\ln(x)} = \underbrace{e^{x\ln(x)}}_{x^x} \cdot \left(\ln(x) + x \cdot \frac{1}{x}\right) =$

Check out graph of x^x!

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Panel 12

Exp. Growth + Decay

Law of natural growth or decay:

rate of change is proportional to amount of stuff

Think: population - the more animals you have, the faster they multiply!

If $y(t)$ is amount of stuff at time t, then

$\frac{dy}{dt} = k \cdot y$ Need y, with $\frac{d}{dt} y = (k) y$

$y = Pe^{kx}$ $\frac{d}{dy} = kPe^{kx} = ky$ $\boxed{y = P_0 e^{kt}}$

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