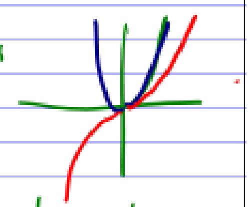
Panel 1

Last Time:

x^2, $\sqrt{x}$

Inverse: f and g are inverse if

$$f(g(x)) = x = g(f(x))$$



When: f has an inverse if

every horiz line intersects graph at most once.

How: Solve $f(x) = y$ for x,

then flip $x \leftrightarrow y$

Derivative: $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

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Panel 2

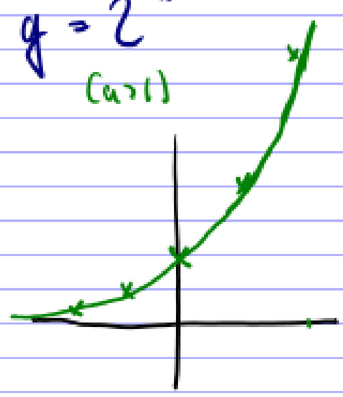
Exponential Function:

$f(x) = a^x$, $a>0$

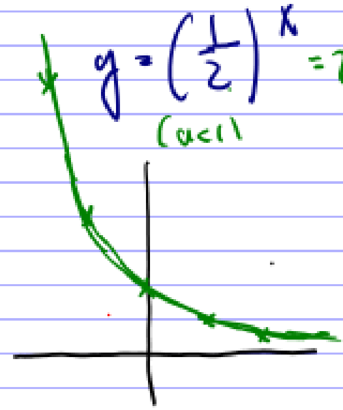
is exp. function with base a.

$y = 2^x$ $(a>1)$

x	2^x
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8



$y = \left(\frac{1}{2}\right)^x = 2^{-x}$ $(a<1)$



x	$\left(\frac{1}{2}\right)^x$
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8

$$\lim_{x\to-\infty} 2^x = 0 \qquad \lim_{x\to\infty} \left(\frac{1}{2}\right)^x = 0$$

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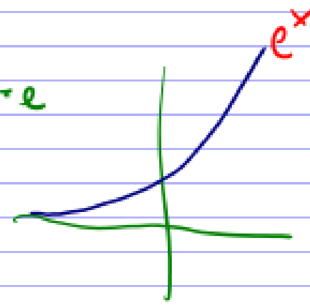
Panel 3

Most common exp. function has base e

Def: Euler's number $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.7182\ldots$

$f(x)=e^x$ is natural exp. function



Ex: $f(0)=1$ $\quad f(1)=e$

$f(2)=e^2 \approx 7.3899\ldots$

$f\left(\frac{3}{5}\right)=e^{3/5}=\sqrt[5]{e^3} \approx 1.822$

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Panel 4

Thm: $f(x)=e^x$ is cont. and diff'ble, and passes the horizontal line test. Thus $f(x)=e^x$ has inverse called natural log. $\quad g(x)=\ln(x)=\log_e(x)$

If $g(x)=a^x$ is arbitrary exp. funct its

$g^{-1}(x)=\log_a(x)$

Basic Relation: $\boxed{y=a^x \iff x=\log_a(y)}$

also: $\log_a(a^x)=x=a^{\log_a(x)}$

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Panel 5

Ex: Find $\log_2(32) = y \iff 2^y = 32 \Rightarrow y = 5$

$\frac{1}{3} = \log_3\left(\sqrt[3]{3}\right) = y \iff 3^y = \sqrt[3]{3} = 3^{1/3} \Rightarrow y$

$\log_5\left(\frac{1}{25}\right) = -2$

$= a^{\log_a(x)} \cdot a^{\log_a(y)} = xy$

$xy = a^{\log_a(xy)} = a^{\log_a(x) + \log_a(y)}$

Properties of Logarithms:

$\log_a(xy) = \log_a(x) + \log_a(y)$ $\quad \left(a^{(\,)}\right.$

$\log_a(x/y) = \log_a(x) - \log_a(y)$

$\log_a(x^p) = p\log_a(x)$

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Panel 6

Find $\log_2(80) - \log_2(5) = \log_2\left(\frac{80}{5}\right) = \log_2(16) = 4$

Solve $2^{5-3x} = 1/8$ $\quad \left|\ \log_2(\ )\right.$

$\cancel{\log_2}\left(\cancel{2}^{5-3x}\right) = \log_2\left(\frac{1}{8}\right)$

$5 - 3x = -3$

$-3x = -8$

$x = 8/3$

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Panel 7

Natural Logarithm: most useful log is

$$y = \ln(x) \quad (= \log_e(x))$$

Ex: Solve $2^{3-5x} = 10 \quad | \ln(\,)$

$$\ln(2^{3-5x}) = \ln(10) \approx 2.3025$$

$$(3-5x)\ln(2) = \ln(10)$$

$$3 - 5x = \frac{\ln(10)}{\ln(2)}$$

$$-5x = \frac{\ln(10)}{\ln(2)} - 3 \quad \Rightarrow \quad x = \frac{3 - \frac{\ln(10)}{\ln(2)}}{5}$$

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Panel 8

Derivative of ln: $\boxed{\frac{d}{dx}\ln(x) = 1/x}$

$$\frac{d}{dx}\ln(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{\ln(x+h) - \ln(x)}{h} =$$

$$= \lim_{h\to 0}\frac{1}{h}\ln\left(\frac{x+h}{x}\right) = \lim_{h\to 0}\frac{1}{x}\frac{x}{h}\ln\left(1+\frac{h}{x}\right) =$$

$$= \lim_{h\to 0}\frac{1}{x}\ln\left(\left(1+\frac{h}{x}\right)^{x/h}\right) = \frac{1}{x}\lim_{h\to 0}\ln\left(\left(1+\frac{1}{\left(\frac{x}{h}\right)}\right)^{\left(\frac{x}{h}\right)}\right)$$

subst: $\frac{x}{h} = n$

$$\frac{1}{x}\ln\left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right) =$$

$$\frac{1}{x}\ln(e) = \frac{1}{x}\cdot 1 = \frac{1}{x}$$

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Panel 9

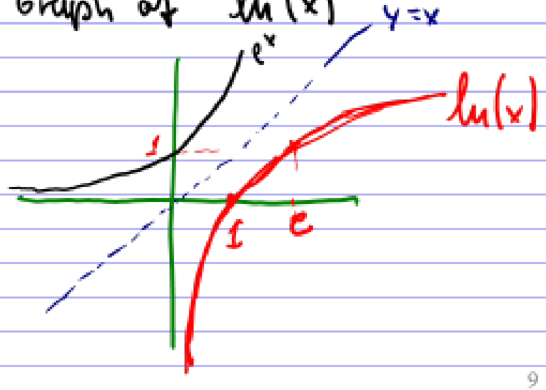
Show that $f(x)=\ln(x)$ is incr. and concave up

$f'(x)=\frac{1}{x}>0 \Rightarrow f$ is increasing , domain of $\log_a(x)$ is $\underline{x>0}$

$\frac{d^2}{dx^2}\ln(x)=\frac{d}{dx}\left(\frac{1}{x}\right)=-\frac{1}{x^2}<0$

$\Rightarrow f$ concave down

Graph of $\ln(x)$



$\log_{(\frac{1}{2})}(x)$

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Panel 10

Find the derivatives of:

$f(x)=\ln(2+\sin(x))$

$g(x)=\ln(x^3+1)$

$h(x)=\ln(\cos(x))$

$k(x)=\sqrt{\ln(1-x)}$

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