Panel 1

Last time

Ex: $f(x) = \tan(x) + x$, near $c=0$, $f(0)=0$

$f'(x) = \sec^2(x) + 1 \Rightarrow f'(0) = 2$

Linear approx: $f(x) \approx f'(c)(x-c) + f(c)$

$\Rightarrow \tan(x) \approx 2\cdot(x-0) + 0 = 2x$

Differentials + Error Propagation

$df = f'(x)\,dx$ relative error $\frac{dx}{x}$ and $\frac{df}{f}$.

MVT + Rolle's Theorem

MVT: $\frac{f(b)-f(a)}{b-a} = f'(c)$. if f is diffble., c exists

Rolle: If $f(a)=f(b)$ then $f'(c)=0$.

Panel 2

Error Propagation.

Say I have a box with height = width, length = 2 width. Measure width as $x = 15$ cm $\pm$ 0.02 cm.

Find error in surface area:

$A = 2x^2 + 4\cdot 2x\cdot x = 10x^2$

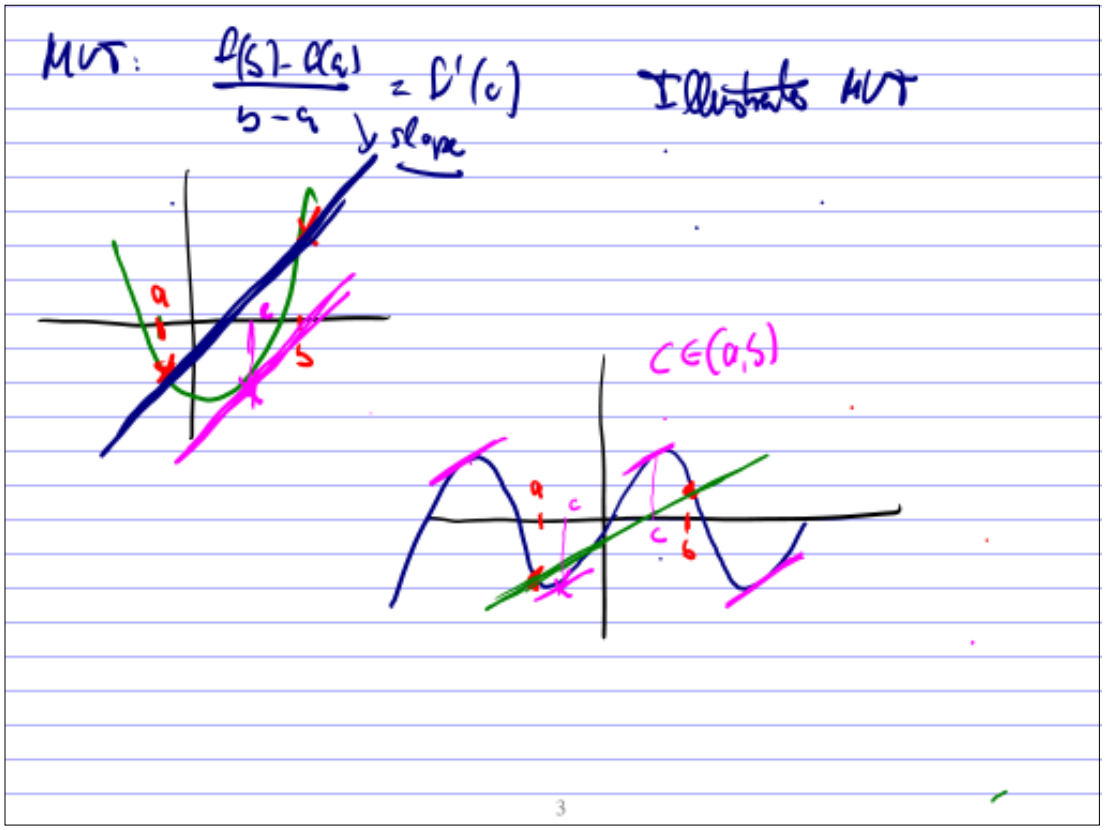
$dA = 20x\,dx = 20\cdot 15\cdot 0.02 = 6$

$A = 10(15)^2 = 2250$

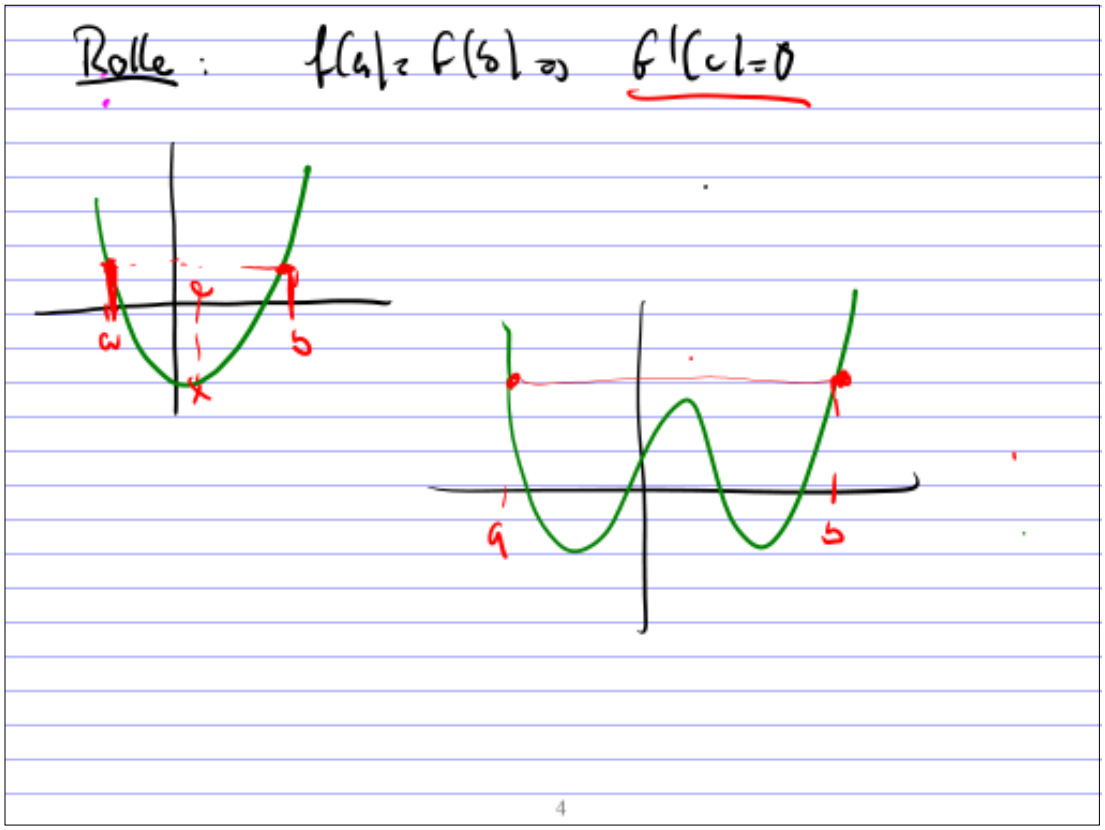
relative error in x is $\frac{0.02}{15} = 0.0013 = 0.13\%$

A is $\frac{6}{2250} = 0.0026 \sim 0.27\%$

Panel 3



Panel 4



Panel 5

Name: _____

Quiz #10

① Find the linear approximation to $f(x) = 2x + \cos(x)$ near $x = 0$

② You measure the side of a cube as 10cm with an error of 0.05 cm, and use it to compute the cube's volume. Find the differential dV as well as the relative error when $x = 10$cm.

$V = x^3$, $dV = 3x^2 dx$ $\quad \frac{dx}{x} = \frac{0.05}{10} = 0.005$

$V(10) = 1000$, $dV = 300 \cdot 0.05 = 15$ $\quad \frac{dV}{V} = \frac{15}{1000} = 0.015$

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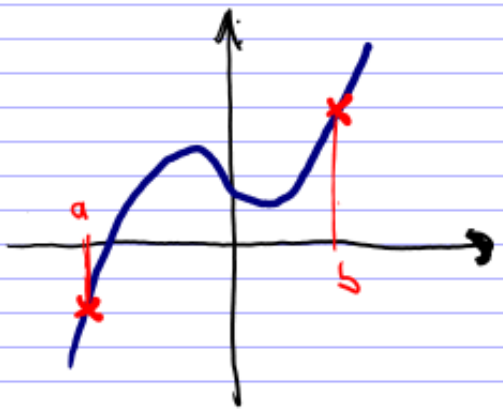
Panel 6

③ The mean value thm (MVT) says that if f is cont. on $[a,b]$ and diffble on (a,b), then there is (at least) one c s.t.

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Find all of these c's in the picture on the right.



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Panel 7

Inverse functions: Check $f(g(x)) = x$ and $g(f(x)) = x$
then g, f are inverse. $g = f^{-1}$
$f = g^{-1}$

Horizontal Line Test: if every horiz. line intersects graph at most once $\Rightarrow$ f has inverse

Thm 1. If f is cont. and has an inverse, then f^{-1} is cont.

Thm 2. If f is diffble and has an inverse, then f^{-1} is diffble and $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$

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Panel 8

Ex: $f(x) = 2x + \cos(x)$. Does f have inverse? Yes, we checked graph horiz. lines.
Find $(f^{-1})'(1)$

Trouble. $y = 2x + \cos(x)$ can't be solved for x!

$f^{-1}(1) = a$ | $f'(x) = 2 - \sin(x)$

$f(f^{-1}(1)) = f(a)$

$1 = f(a) = 2a + \cos(a) \Rightarrow a = 0$ by guessing

$\Rightarrow (f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)} = \frac{1}{2}$

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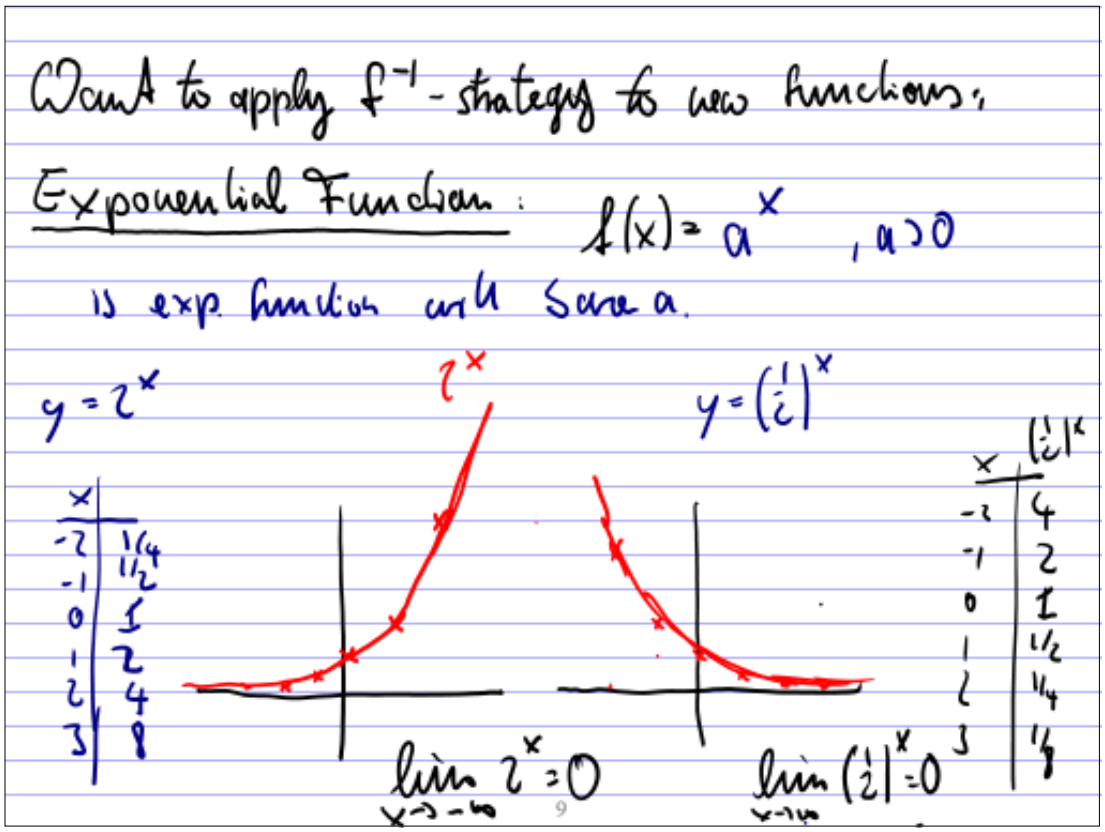
Panel 9


Want to apply f^{-1} - strategy to new functions:
Exponential Function: $f(x) = a^x$, $a > 0$
is exp. function with base a.
$y = 2^x$
2^x
$y = (\frac{1}{2})^x$
$(\frac{1}{2})^x$
$\lim_{x \to -\infty} 2^x = 0$
$\lim_{x \to \infty} (\frac{1}{2})^x = 0$