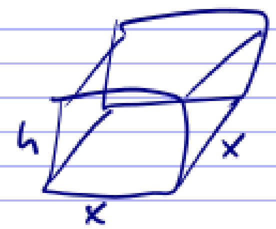
Panel 1

If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.



x = length of bottom

h = height

Know: $1200 = x^2 + 4xh$

Max $V = x^2 h = x^2 \cdot \left(\frac{1200 - x^2}{4x} \right) =$

$= \frac{1}{4}(1200x - x^3)$

$V'(x) = \frac{1}{4}(1200 - 3x^2) = \frac{3}{4}(400 - x^2)$

$\boxed{x = 20}$

1

Panel 2

Last Time

- Optimization ✓
- Implicit differentiation

$$y' = \frac{2y - 4x^3}{4y^3 - 2x}$$

Ex: $x^4 + y^4 = 2xy$

Say $y = y(x)$. Find $\frac{dy}{dx}$: $\frac{d}{dx}(x^4 + y^4) = \frac{d}{dx}(2xy)$

$4x^3 + 4y^3 y' = 2y + 2x y'$

Say $x = x(y)$ Find $\frac{dx}{dy}$: $\frac{d}{dy}(x^4 + y^4) = \frac{d}{dy}(2xy)$

$4x^3 \cdot x' + 4y^3 = 2x' y + 2x \cdot 1$

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Panel 3

Ex: Suppose $x^2y^3 = x^3 + y^2$. Assume that both x and y are functions of t. Find $x'(t)$.

$x = x(t), \quad y = y(t)$

$$\frac{d}{dt}\left(x^2 y^3\right) = \frac{d}{dt}\left(x^3 + y^2\right)$$

$$2(x)x' y^3 + \underline{x^2 \cdot 3y^2 \cdot y'} = 3x^2 x' + \underline{2yy'}$$

$$\boxed{x' = \frac{2yy' - x^2 3y^2 y'}{2xy^3 - 3x^2}}$$

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Panel 4

Ex: Find y'' if $x^4 + y^4 = 16$ $\quad (y = y(x))$

$$\frac{d}{dx}\left(x^4 + y^4\right) = \frac{d}{dx}16$$

$$\cancel{4}x^3 + \cancel{4}y^3 y' = 0$$

$$x^3 + y^3 y' = 0$$

$$\frac{d}{dx}\left(x^3 + y^3 y'\right) = \frac{d}{dx}(0)$$

$$3x^2 + 3y^2 \cdot y' y' + y^3 y'' = 0$$

$$3x^2 + 3y^2 (y')^2 + y^4 y^3 = 0$$

$$y' = -\frac{x^3}{y^3}$$

$$y'' = -\frac{3x^2 y^3 - x^3 3y^2 \cdot y'}{y^6}$$

Panel 5

Consider the equation $x^2 + xy = 4$

$x = x(y)$: $\frac{d}{dy}\left[x^2 + xy\right] = \frac{d}{dy}(4)$

$2x x' + x' y + x \cdot 1 = 0$

$y = y(x)$: $\frac{d}{dx}\left(x^2 + xy\right) = \frac{d}{dx}(4)$

$2x + 1y + x \cdot y' = 0$

$x = x(t), y = y(t)$: $\frac{d}{dt}\left(x^2 + x \cdot y\right) = \frac{d}{dt}(4)$

$2x \cdot x' + x' y + x \cdot y' = 0$

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Panel 6

Appl. of Implicit Differentiation : Related Rates

Ex: Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm³/sec. How fast is the radius increasing if the diameter is 50 cm.

Derivative is "Rate of change"

$V = \frac{4}{3}\pi r^3$ $V = V(t),\ r = r(t)$

$\frac{d}{dt}V = \frac{d}{dt}\frac{4}{3}\pi r^3$ Know: $V' = 100$, $r' =$ want when $r = 25$ cm

$V' = 4\pi r^2 \cdot r' \Rightarrow 100 = 4\pi(25)^2 \cdot r' \Rightarrow r' = \frac{100}{4\pi(25)^2}$

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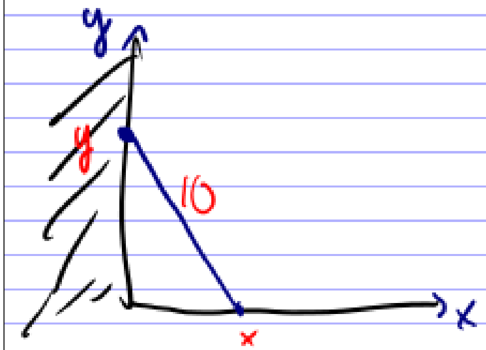
Panel 7

Ex: A 10 foot ladder rests against a wall. The base of the ladder slides away from the wall at a rate of 1 m/sec. How fast is the top sliding down when the bottom is 6 feet away from the wall.



Know: $\frac{dx}{dt} = 1$, $\frac{dy}{dt} = ?$

$$x^2 + y^2 = 100 \quad \Big| \frac{d}{dt}$$

$$\cancel{2}xx' + \cancel{2}yy' = 0$$

If $x = 6 \Rightarrow 36 + y^2 = 100$

$y = 8$

$$y' = -\frac{xx'}{y} = -\frac{6 \cdot 1}{8} = \underline{-\frac{3}{4}}$$

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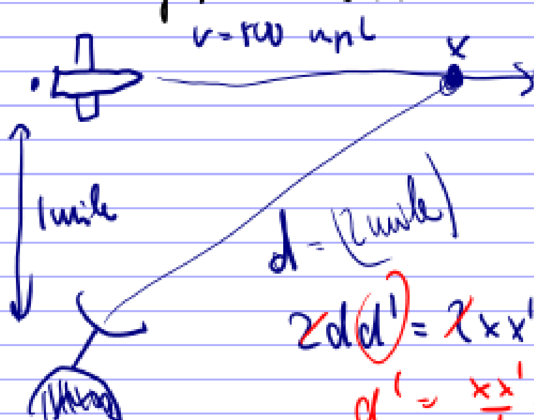
Panel 8

Ex: A plane flying horizontally at an altitude of 1 mile and a speed of 500 mph directly over a radar station. Find the rate at which the distance from plane to radar station is increasing when it is 2 miles away from the station.

$x = \sqrt{3} \Leftarrow (2)^2 = x^2 + 1$



Know: $d^2 = x^2 + 1$

d/dt

$\frac{dx}{dt} = 500$

$\frac{dd}{dt}$? Want.

$$2dd' = 2xx'$$

$$d' = \frac{xx'}{d} = \frac{\sqrt{3} \cdot 500}{2} = \boxed{433 \text{ mph}} \; ?$$

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Panel 9

Linearization

Recall. $\lim_{x \to c} \frac{f(x) - f(c)}{x - c} = f'(c)$

$\Rightarrow \frac{f(x) - f(c)}{x - c} \simeq f'(c) \Rightarrow f(x) - f(c) \simeq f'(c)(x-c)$

$$\Rightarrow \boxed{f(x) \simeq f'(c)(x-c) + f(c)}$$

Ex: $f(x) = \sin(x)$ Find linearization of $\sin(x)$ near 0.

$\Rightarrow \sin(x) \simeq \cos(0)(x-0) + \sin(0) = x$

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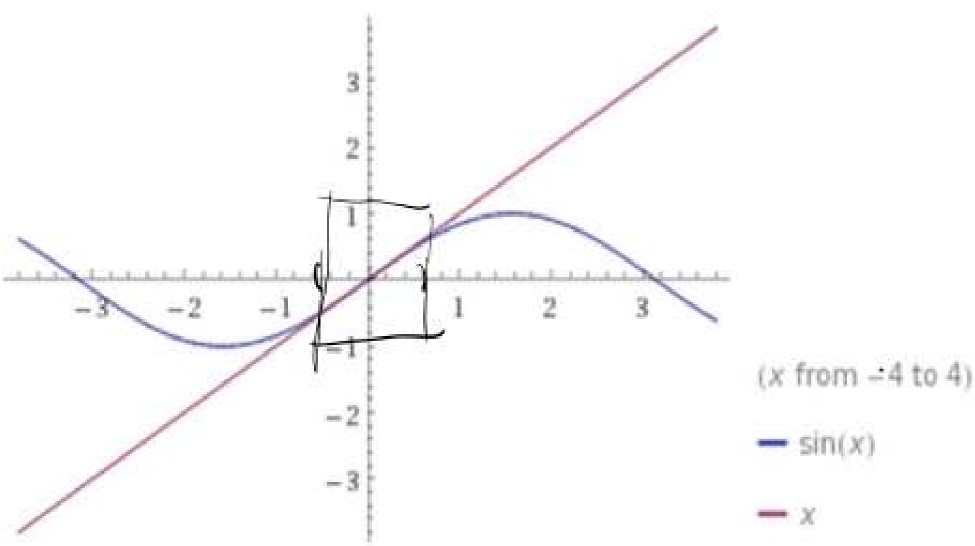
Panel 10

$\sin(x) \simeq x$ near $x = 0$

Ex: $\sin(0.1) \simeq 0.1$



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Panel 11

Ex: Use linearization of $f(x) = \sqrt{x+3}$ at $c=1$ to approximate $\sqrt{3.98}$ and $\sqrt{4.05}$

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Panel 12

Ex: Use linearization of $f(x) = \sqrt{x+3}$ at $c=1$ to approximate $\sqrt{3.98}$ and $\sqrt{4.05}$

$f(x) \approx f'(c)(x-c) + f(c)$

$f(x) = \sqrt{x+3} \Rightarrow f'(x) = \frac{1}{2}(x+3)^{-1/2} \cdot 1$

$c=1$: $f(1) = \sqrt{4} = 2$, $f'(1) = \frac{1}{2}\frac{1}{2} = 1/4$.

$\sqrt{x+3} \approx \frac{1}{4}(x-1) + 2$

$\sqrt{3.98} \approx \frac{1}{4} \cdot (-0.02) + 2 = 2 - 0.005 = 1.995$

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Panel 13

Quiz

(1) Opt. problem (max/min)

(2) Implicit diff

(3) Related rates

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